IN COOPERATION WITH ib

MYP Mathematics

A concept-based approach

ENHANCED ONLINE

Year 2

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Contents

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1 Ratios and proportions

In this unit, you will use ratios and proportions to understand how humans compete and cooperate with one another. However, ratios and proportions can also be useful in other, surprising contexts. Things that we often take for granted, like the fairy tales we heard growing up or the politics that we hear about every day, can also be analysed and understood through these important mathematical tools.

Fairness and development

Politics and government

There are many different forms of government, from monarchies to democracies to republics. Democratic countries may use a parliamentary system of government or a presidential one. What do these terms mean?

Is one system more representative of the citizens than another? Do the different systems represent a country's people equally, despite their differences? What is the ratio of representatives to citizens? Is this ratio the same in urban areas and rural areas? Is it possible that some people have no representation at all?

Personal and cultural expression

Does magic know math?

Traditional tales are a part of growing up all over the world. What if those stories were actually true? Is that possible? A look at popular fables through the lens of ratios and proportions might reveal some flaws in their logic. Would the morals of these stories be more believable if the stories were as well?

Hansel and Gretel is a story written by the Brothers Grimm about a young brother and sister. Alone in the woods, they encounter a witch who entices them to come with her, all in the hope of eating them!

The witch's cottage is made of gingerbread and candy, which tempts the children in. How would a typical gingerbread recipe have to be adapted to create a cottage large enough to live in? How much would those ingredients cost?

Cinderella tells the story of a girl who meets a prince at a ball. She has to leave before her carriage turns back into a pumpkin and her horses into mice and, in the rush, she leaves behind a glass slipper. The prince visits every girl in the kingdom to find the owner of the slipper. Could he have used proportions to narrow down his search?

When the fairy godmother transforms a pumpkin into a carriage and mice into horses, are they all enlarged using the same ratio? Does magic know math or is it really just luck?

1 Ratios and proportions

Competition and cooperation

KEY CONCEPT: LOGIC

Related concepts: Equivalence, Quantity and Simplification

Global context:

Part of studying the global context **identities and relationships** involves looking at how humans cooperate and compete with one another. Ratios and proportions allow you to analyze these human interactions and even make decisions about what may be fair or desirable. There are also times when competitors are forced to cooperate with one another, blurring the lines between who may be a friend and who may be a foe.

Statement of Inquiry:

Using a logical process to simplify quantities and establish equivalence can help analyse competition and cooperation.

Objectives

- Defining and simplifying ratios
- Dividing a quantity in a given ratio
- Defining proportion and demonstrating proportional relationships
- Representing proportional relationships using tables, equations and graphs
- Finding the constant of proportionality for a proportional relationship
- Applying mathematical strategies to solve problems using proportional reasoning

Inquiry questions

F
What does it mean to simplify?
What does it mean to be equivalent?

C
How can you establish equivalence?
How are simplification and equivalence related?

D
What makes for healthy and fair competition?
Which is more about being equal, competition or cooperation? Explain.

ATL1 Organization skills

Create plans to prepare for summative assessments (examinations and performances)

ATL2 Affective skills

Practise positive thinking

You should already know how to:

- round numbers correctly
- solve problems involving percentages
- convert between different units
- solve simple equations

Introducing ratio and proportion

Competition among humans is a very natural part of who we are. There are professional sports teams who compete with each other, as well as the adolescents who are hoping to be the next great athlete. The Olympics bring together athletes from around the world for two weeks of competition. There is also the day-to-day competition among people seeking that exciting new job.

Despite this constant competition, humans must also cooperate with one another. Those same athletes and those same employees must also find a way to work *with* others to accomplish group goals.

Where is the line between competition and cooperation? Can it be analysed? In this unit, you will learn to describe competition and cooperation among humans and, hopefully, understand where that line is.

Reflect and discuss 1

In a small group, answer the following:

- Give two examples of times when you were competitive.
- Give two examples of times when you were cooperative.
- Which do you experience more, competition or cooperation? Explain.
- Has there ever been a time when you competed against someone with whom you later had to cooperate? If so, describe how that went.

Ratios

In mathematics, a *ratio* is a way to compare different quantities that are measured using the same units. Just like fractions, ratios can be simplified.

For example, if there are 12 girls and 8 boys on your team, the ratio of girls to boys would be written as 12 to 8 or 12 : 8 (also read as ‘12 to 8’).

In simplified form, this would be written as 3 to 2 or 3 : 2. Because the units are the same for both quantities (people), we do not need to write them.

The ratio of girls to the total number of students would be written as 12 to 20, or 12 : 20. In simplified form, this would be written as 3 : 5.

The ratio of boys to girls to the total number of students would be written as 8 to 12 to 20, or 8 : 12 : 20. In simplified form, this would be written as 2 : 3 : 5.

Investigation 1 – Simplifying ratios

criterion B

a Copy the table below.

Unsimplifed ratio	Simplified ratio
8 : 12	
21 : 35	
18 : 9	
4 : 16	
15 : 35	
45 : 10	
36 : 42	

1 : 4   9 : 2   2 : 3   7 : 3   3 : 7   6 : 7   3 : 5   5 : 6   2 : 1   1 : 9   8 : 3   6 : 5   5 : 4

b Find the simplified form of each ratio in the table from among the options on the right. Write them in the table next to the unsimplified ratio.

c Explain how you figured out which ratios belong together.

d Generalize the procedure to simplify a ratio.

e Verify your procedure with two examples different than the ones above.

Reflect and discuss 2

- Can all ratios be simplified? Explain.
- How is simplifying ratios similar to simplifying fractions? Explain.

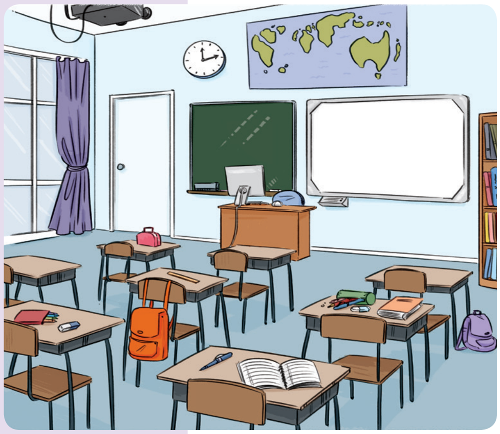
Pairs

Activity 1 – Ratios around the room

1 With your partner, identify some ratios that you see around your classroom. Remember that a ratio is a comparison of two quantities with the same units.

2 With your partner, play **I Spy**. The first player looks around the room and finds two quantities that can be expressed as a ratio. The player says, "I spy with my little eye a ratio of ____ ", filling in the blank with a ratio. The second player tries to guess what the two quantities represent. When the second player is successful, the roles are switched.

3 Play **I Spy** four times each, with the last round using ratios that have to be simplified.



Reflect and discuss 3

In pairs, answer the following questions:

- Can you make a ratio out of any two quantities? Explain.
- Is a ratio of 1 : 2 the same as a ratio of 2 : 1? Explain.

Two ratios that can be simplified to the same ratio are said to be *equivalent*.

For example, these three ratios:

12 : 16
3 : 4
21 : 28

are all equivalent because they all simplify to 3 : 4.

Investigation 2 – Equivalent ratios

Criterion B

1 Copy the following table.

Ratio	Equivalent ratios				Simplified ratio
4 : 6	40 : 60	12 : 18	24 : 36	36 : 54	
30 : 12	60 : 24	15 : 6	300 : 120	120 : 48	
10 : 40	5 : 20	30 : 120	80 : 320	2 : 8	
8 : 28	16 : 56	80 : 280	32 : 112	40 : 140	
16 : 8	8 : 4	48 : 24	4 : 2	320 : 160	
14 : 20					
36 : 6					

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2 Find the simplified form of each ratio and write it in the right-most column.

3 Write down how each equivalent ratio was found, given the ratio in the left-most column.

4 In the middle column, write down another ratio equivalent to those given.

5 Complete the last two rows of the table by writing in four equivalent ratios each.

6 Write down how to find a ratio that is equivalent to a given ratio.

7 Verify your procedure with two examples that are different than the ones in the table.

Reflect and discuss 4

- How are equivalent ratios like equivalent fractions? Explain.
- Knowing that ratios generally contain whole numbers, how would you simplify the ratio 0.5 : 3?

Example 1

Q One of the most famous competitions in the 1970s was the 'Battle of the Sexes' tennis match between Billie Jean King and former tennis champion Bobby Riggs.
Riggs challenged all female tennis players to defeat him, which King did in 1973. The ratio of games won was 18 : 10 in favor of King. Give three ratios equivalent to 18 : 10.

A 18 : 10 = 9 : 5

Divide each quantity by 2.
Note that this is now a simplified ratio since neither quantity can be divided further.

18 : 10 = 36 : 20

Multiply each quantity in the original ratio by any whole number, in this case 2….

18 : 10 = 90 : 50

…and in this case, 5.

Three ratios equivalent to 18 : 10 are
9 : 5, 36 : 20 and 90 : 50.

Equivalent ratios can be very helpful when trying to solve a wide range of problems involved in both cooperation and competition.

Example 2

Q A marathon is a race of approximately 42 km. In many marathon competitions, there is a relay category in which a team of runners works together to complete the race, with each competitor running some portion of the entire distance.

a Suppose three friends create a relay team and, because of their differing abilities, they decide to run distances in a ratio of 1 : 2 : 3. How far does each person run?

b After the first 2 km, the first runner injures himself. The remaining two runners decide to run the rest of the race in a ratio of 3 : 5. How far did the remaining two competitors run?

A **a** $1 + 2 + 3 = 6$

Find the total of the number of terms in the ratio.

Runner 1's portion $= \frac{1}{6}$

Express each runner's contribution as a fraction of the total.

Runner 2's portion $= \frac{2}{6}$ or $\frac{1}{3}$

Runner 3's portion $= \frac{3}{6}$ or $\frac{1}{2}$

Runner 1's portion $= \frac{1}{6}$ of 42, or $\frac{1}{6} \times 42 = 7$

Calculate the fraction of the race run by each runner.

Runner 2's portion $= \frac{1}{3}$ of 42, or $\frac{1}{3} \times 42 = 14$

Runner 3's portion $= \frac{1}{2}$ of 42, or $\frac{1}{2} \times 42 = 21$

7 km + 14 km + 21 km = 42 km ✓

Check your solution to make sure it works.

One runner will run 7 km, another 14 km and the third one 21 km.

b Find a ratio equivalent to 3 : 5

At the 2 km mark, there are 40 km remaining in the race. The goal is to find a ratio equivalent to 3 : 5 where the values sum to 40.

$3r : 5r$

$3r + 5r = 40$

Thus, if you multiply each of the values by the same constant, then that should sum to 40.

$8r = 40$

$r = 5$

Simplify the left-hand side and solve.

Continued on next page

$3r = 15$ km
$5r = 25$ km

Substitute the value for r back into the ratio $3r : 5r$ to find the distance each person ran.

15 km + 25 km = 40 km ✓

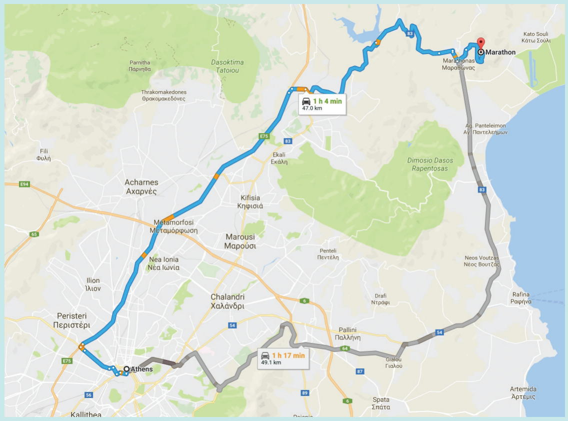
Check your solution to make sure it works.

One runner will run 15 km and the other will run 25 km.

Did you know?

The legend goes that Pheidippides, a Greek messenger, ran 25 miles from Marathon to Athens, to deliver the news of the Athenians' victory over the Persians in 490 B.C. The modern marathon distance became 26.2 miles at the 1908 Summer Olympics in London, where the marathon course was designed so that it could start at Windsor Castle and finish at the Olympic stadium.

The oldest marathon in the United States is the Boston Marathon, which has been run continually since 1897.



Practice 1

1 Simplify the following ratios. If a ratio is already in its simplest form, write the word 'simplified'.

a 3 : 5 **b** 4 : 6 **c** 12 : 16

d 100 : 200 **e** 700 : 35 **f** 0.4 : 6

g 9 : 10 **h** 1 : 6 **i** 950 : 50

j 13 : 200 **k** 1006 : 988 **l** 64 : 16

2 Write three equivalent ratios for each of these ratios:

a 2 : 5 **b** 2 : 18 **c** 9 : 27 **d** 10 : 25

e 6 : 7 **f** 15 : 45 **g** 8 : 1 **h** 12 : 3

3 DC and Marvel created comics with teams of superheroes in order to compete for readers. DC had 'The Justice League' while Marvel had 'The Avengers'. The ratio of the number of original members for the respective superhero teams was (DC to Marvel) 7 : 5.

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a Over a period of several years, each company added new superheroes to their team. If the ratio of the number of superheroes was kept the same over that period, give two examples for the number of superheroes that could have been in each team.

b Some time later, the ratio of superheroes (DC to Marvel) was 21 : 10. Is this ratio equivalent to the ratio in part **a**? Show your working.

c At one point, The Justice League had 91 superhero members. How many superheroes would you expect the Avengers to have if the original ratio was maintained? Show your working.

4 Draw a picture that includes all of the following elements:

- a 2 : 3 : 5 ratio of green flowers to blue flowers to red flowers
- boys and girls in a ratio of 1 to 2
- trees and bushes in a ratio of 1 : 1.

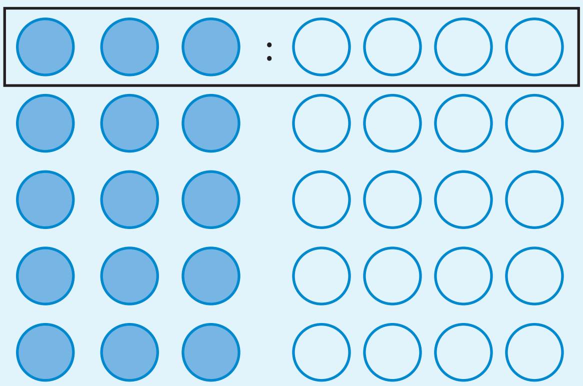
You may add whatever else you like to your picture to make it unique.

5 From the following list, write down all of the ratios that are equivalent to each other.

4 : 5 2 : 8 1 : 3 1 : 4 12 : 15 5 : 10

4 : 8 4 : 12 9 : 18 12 : 48 28 : 35 7 : 21

6 Use the diagram here to justify why the ratio 3 : 4 is equivalent to the ratio 15 : 20.



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7 In the Pokémon card game, two players enter into friendly competition to 'battle' each other in a field and knock out each other's Pokémon. A starting deck of cards could have a ratio of Pokémon cards to trainer cards to energy cards of 5 : 2 : 3. If the deck contains 60 cards, how many of each card type will there be?

8 Men and women often do the same job, though they do not always get the same pay. When men and women get different pay for the same work, there is a *gender pay gap*. The ratio of women's wages to men's wages for four countries is given in this table.

Country	Women's wages : Men's wages for same work (currency)	Simplified ratio
Bulgaria	1.5 : 2 (lev)	
Australia	0.83 : 1 (dollar)	
Russia	2.40 : 3 (ruble)	
United Arab Emirates	0.99 : 1 (dirham)	

a Complete the table by finding a simplified ratio for each country.

b Which country has the largest gender pay gap? Explain.

c In which country does the gender pay gap disadvantage women the most? In which country does the gender pay gap disadvantage women the least? Explain.

9 Given the following ratios and totals, find the portion represented by each part of the ratio.

a ratio = 3 : 7 , total = 90

b ratio = 1 : 2 : 5 , total = 88

c ratio = 9 : 3 : 4 , total = 32

d ratio = 2 : 4 : 5 : 9 , total = 120

10 In a regional mathematics competition, the ratio of Swedish students to Danish students to Finnish students is 14 : 22 : 13. Altogether, there are 1029 students. Find how many students come from each of the three countries. Show your working.

11 Given that the ratios in each part below are equivalent, find what quantity each variable represents.

a $1 : 4 = 2 : k$

b $2 : 5 : 8 = 16 : p : 64$

c $8 : 7 = 24 : x$

d $2 : 3 : 7 = 14 : r : s$

e $2 : 7 : 10 = 1 : q : 5$

f $4 : 1 : 12 = 6 : z : 18$

ATL2

Reflect and discuss 5

Reflect and discuss 5

- In terms of your learning, write down what is going well so far in this unit.
- Describe a time when thinking positive thoughts made a difference.

Other ways to represent ratios

Ratios can be represented in a variety of forms and you should feel comfortable moving between these different representations. How to do that is the focus of the next activity.

Activity 2 – Competition in government

Becoming an elected official usually involves competing against other men and women for the same position. Once elected, these officials must then cooperate with people from different political parties and viewpoints in order to effect change. In this activity, you will explore the ratio of women to men that hold elected positions in the governments of various countries.

Below are data showing the number of elected seats held by men and women around the world.

Country	Women	Men	Total elected officials	Ratio of women to men
Argentina	130	199	329	
Australia	73	153	226	
Canada	88	250	338	
France	223	354	577	
Sweden	152	197	349	
United Kingdom	191	459	650	
United States	104	431	535	

1 Make a copy of the table and write the ratio of women in government to men in government for the seven countries.

2 Which country has the highest ratio of women to men? Which has the lowest? How can you tell?

3 Now, you are going to use a decimal and a percentage to help compare the quantity of women in government positions around the world. Fill in a table like the following to help organize your information.

Continued on next page

Country	Ratio of women out of **total members** of government expressed as a fraction	Ratio of women out of total members of government expressed as a decimal	Ratio expressed as a percentage (of women out of total members of government)
Argentina			
Australia			
Canada			
France			
Sweden			
United Kingdom			
United States			

4 What does a percentage mean as a ratio? Explain.

5 What are the advantages of using percentages to represent ratios?

6 In 1991, Argentina adopted the world's first gender quota law. The law mandates that political parties nominate women for at least 30% of the electable positions on their candidate list. Which of the countries above have at least 30% of women in government?

Reflect and discuss 6

- Explain a general rule for converting a ratio to a percentage.
- Would it be possible to represent the ratio 3 : 5 : 11 as a fraction or a percentage? Explain.
- Describe the advantages and disadvantages of representing a ratio as a fraction and as a percentage.

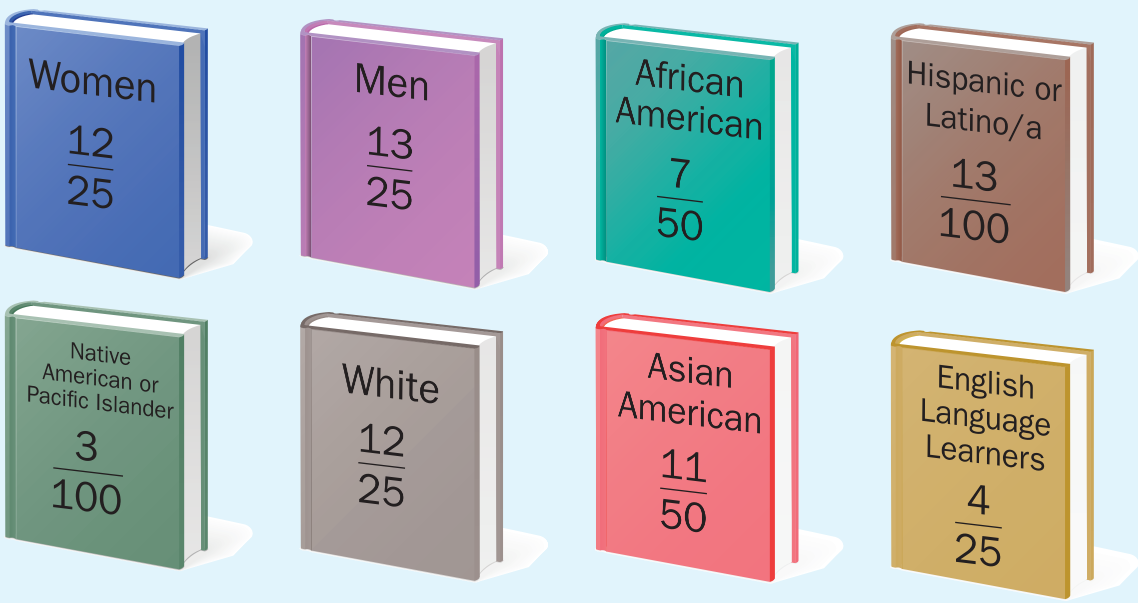
Practice 2

1 Match each ratio card with a fraction card **and** a percentage card.

Ratio cards		Fraction cards		Percentage cards	
112 : 564	42 : 70	$\frac{5}{12}$	$\frac{4}{5}$	80%	19.9%
24 : 30	23 : 46	$\frac{3}{5}$	$\frac{1}{2}$	50%	41.7%
2.5 : 6	17.5 : 21	$\frac{1}{3}$	$\frac{28}{141}$	83.3%	11.1%
8 : 72	18 : 54	$\frac{5}{6}$	$\frac{1}{9}$	33.3%	60%

2 Universities and colleges around the world use a selective admissions process to build a diverse population of students. Students of all backgrounds compete with one another for the limited number of places in these institutions of higher education.

For a top university in North America, these are approximate ratios for the number of students admitted from a variety of groups compared with the whole school population.



a What do these fractions mean? Explain using an example.

b Represent these quantities as a ratio, decimal and percentage. Which form is most effective to represent the diversity of the school? Explain.

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c Show that the number of women and men admitted to the university is approximately equal. Explain which representation allows you to see this most easily.

3 The Summit Series was a famous hockey competition between Canada and the USSR during the Cold War. In 1972 and again in 1974, the Soviet team played against Canadian all-stars from the National Hockey League in an 8-game series. Half of the games were played in Canada and half in Moscow. Players on Team Canada, who would normally play against one another, now joined forces to represent their country.

The results of the 1972 series are given below.

Played in Canada

Game	Score
1	USSR 7 – Canada 3
2	Canada 4 – USSR 1
3	Canada 4 – USSR 4
4	USSR 5 – Canada 3

Played in the Soviet Union

Game	Score
1	USSR 5 – Canada 4
2	Canada 3 – USSR 2
3	Canada 4 – USSR 3
4	Canada 6 – USSR 5

a Write the ratio of games won to total number of games played for each team. Represent the quantities in four different ways (ratio, fraction, decimal, percentage).

b Which representation is the most effective when trying to describe who won the series? Explain.

c Write the ratio of goals scored by the Soviet Union to the number of goals scored by Team Canada. Express this ratio in four different ways.

d Which representation in part **c** is most effective? Explain.

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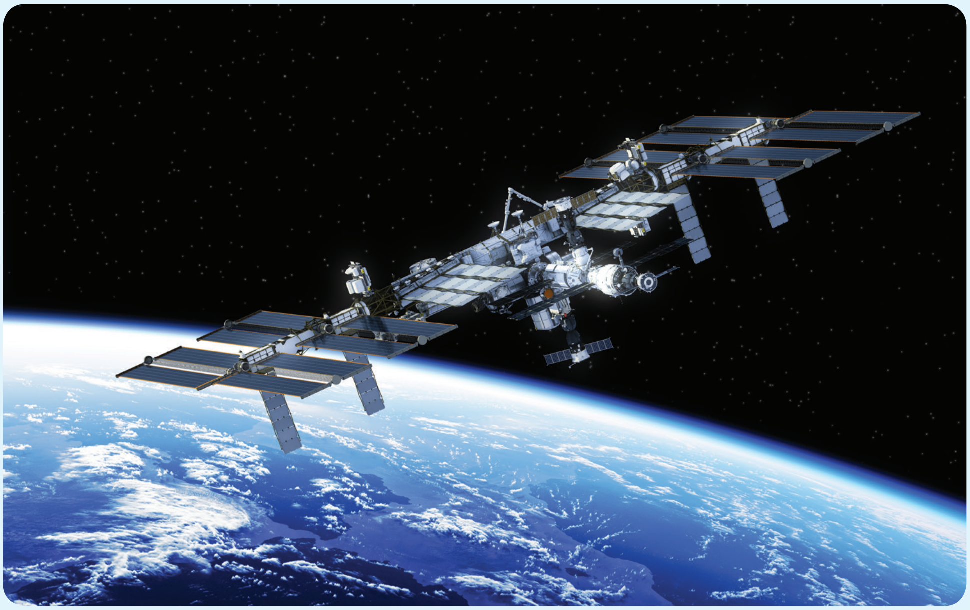
4 The International Space Station (ISS) is a habitable station orbiting Earth several times each day. It is a joint project of five space agencies from Europe, Canada, Japan, Russia and the United States and it has been continuously inhabited since the year 2000. Its goal is to help conduct scientific research and possibly be a stopping point for future trips to the Moon or Mars.



WEB LINK

a Go online to find the current astronauts aboard the ISS. Represent the ratio of the number of astronauts from each country in four different ways. Which representation is the most effective? Explain.

b Expedition 42 had a ratio of women to men of 2 : 4. Represent the ratio of women to the total number of astronauts as a percentage.

c It is very expensive to bring water to the station, so much of the water consumed by astronauts is recycled from a variety of sources. The ratio of the amount of water used for food (for one astronaut) to the amount of water one astronaut drinks is 21 : 43. Find the percentage of total water that is used for drinking (assuming that water is not used for any other purposes).

d Because the ISS orbits the Earth several times each day, astronauts see multiple sunrises and sunsets. An "ISS day" is 6.25% of an Earth day. Represent the ratio of an ISS day to an Earth day as a fraction in its simplest form. Hence find the number of times the ISS orbits Earth in one day.

Formative assessment

criterion C, D

Humans have created competitions in a wide range of activities. Some involve time, some involve measurements and others are judged. Every summer since 1972, participants from around the world have competed in a hotdog-eating contest on Coney Island, New York. In this event, competitors have to eat as many hotdogs as they can in a given time.

In 2006, men and women competed against each other. The competition was 12 minutes long and produced the results in Table 1.

Cash prizes were awarded beginning in 2007. The male and female winners each receive US$10 000.

Table 1

Position	Competitor	Hotdogs eaten	Competitor's weight (kg)
1	Takeru Kobayashi	53.75	58
2	Joey Chestnut	52	104
3	Sonya Thomas	37	44

In 2017, competition was based on gender and lasted only 10 minutes. The men's results are in Table 2.

Table 2

Position	Competitor	Hotdogs eaten	Prize money (US$)
1	Joey Chestnut	72	10 000
2	Carmen Cincotti	60	5 000
3	Matt Stonie	48	2 500

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The 2017 women's results are shown in Table 3.

Table 3

Position	Competitor	Hotdogs eaten	Prize money (US$)
1	Miki Sudo	41	10 000
2	Michelle Lesco	32.5	5 000
3	Sonya Thomas	30	2 500

a Write down the ratio of prize money to the number of hotdogs eaten for each of the top three male and female competitors in 2017.

b Simplify this ratio and then express it in four different ways (a : b, fraction, decimal, percentage).

c Which representation is most useful? Explain.

d Represent the ratio of the number of hot dogs eaten by each of the winners (from all of the competitions) to the duration of the competition in four ways.

e Which representation in part **d** makes Joey Chestnut's world record in 2017 most visible? Explain.

f Use a ratio to show that the results of the 2006 contest could have been different. Make sure that your ratio **clearly** demonstrates why the results could have been different. Would this be a fair way to determine the winner that could include both genders? Justify your answer.

g Is this competition more fair now that the contest is divided by gender? Explain.

ATL2

Reflect and discuss 7

- Write down what is going well so far in this unit in terms of your learning.
- Describe something positive about yourself that you know you can always rely on in class.

Proportions

Two equivalent ratios form a *proportion*, such as the following:

$$\frac{2}{15} = \frac{6}{30} \quad \text{or} \quad 2 : 15 = 6 : 30$$

Proportions and proportional reasoning can be extremely helpful in solving problems.

Solving proportions

You have previously worked with equivalent fractions. An equation showing that two fractions or ratios are equivalent is called a proportion. These proportions contain patterns that you will discover in the next investigation.

Investigation 3 – Solving proportions

criterion B

Look at the following equivalent fractions/proportions.

$\frac{2}{3}=\frac{4}{6}$	$\frac{10}{20}=\frac{4}{8}$	$\frac{1}{6}=\frac{4}{24}$	$\frac{1}{4}=\frac{10}{40}$
$\frac{10}{15}=\frac{2}{3}$	$\frac{5}{6}=\frac{10}{12}$	$\frac{3}{4}=\frac{6}{8}$	$\frac{2}{5}=\frac{6}{15}$
$\frac{3}{8}=\frac{12}{32}$	$\frac{2}{7}=\frac{6}{21}$	$\frac{4}{9}=\frac{20}{45}$	$\frac{100}{250}=\frac{4}{10}$

a Write down patterns you see that are the same for **all** pairs.

b Share the patterns you found with a peer. Which one(s) can be written as an equation?

c Generalize your patterns by writing one or more equations based on the proportion $\frac{a}{b}=\frac{c}{d}$.

d Add four more equivalent fractions/proportions to the ones here and verify your equations for these new proportions.

Reflect and discuss 8

- How could you use your equation(s) to find the missing value in each of the following?

  $\frac{3}{8}=\frac{x}{96}$ $\quad\frac{m}{11}=\frac{75}{165}$ $\quad\frac{2.4}{x}=\frac{8.4}{20.5}$

- Explain how converting a ratio represented as a fraction to a percentage involves a proportion.

Example 3

Q Find the missing value in each of these proportions. Round your answers to the nearest hundredth where necessary.

a $\frac{8}{22}=\frac{12}{x}$ **b** $\frac{5}{9}=\frac{x}{17}$

A **a** If $\frac{8}{22}=\frac{12}{x}$,

then $8x = 12 \times 22$

In a proportion, the products of the quantities on each diagonal are equivalent.

$8x = 264$

Simplify the right-hand side

$\frac{8x}{8}=\frac{264}{8}$

Solve the equation by dividing each side by 8.

$x = 33$

The missing value is 33.

b If $\frac{5}{9}=\frac{x}{17}$,

then

$9x = 5 \times 17$

In a proportion, the products of the quantities on each diagonal are equivalent.

$9x = 85$

Simplify the right-hand side

$\frac{9x}{9}=\frac{85}{9}$

Solve the equation by dividing each side by 9.

$x = 9.\dot{4}$

The missing value is approximately 9.44.

Activity 3 – Solving proportions: a different approach

Another way to solve proportions uses the techniques you learned to solve equations.

1 Solve the following equations.

$$\frac{w}{4}=5 \qquad 10=\frac{y}{3} \qquad \frac{x}{3}=\frac{21}{9} \qquad \frac{5}{8}=\frac{h}{16}$$

2 Explain how you solved them. Justify your process.

3 Given the equivalent fractions below, find the reciprocal of each.

$$\frac{5}{10}=\frac{1}{2} \qquad \frac{3}{4}=\frac{12}{16} \qquad \frac{24}{18}=\frac{8}{6} \qquad \frac{10}{35}=\frac{2}{7}$$

4 What do you notice about your answers? Are the new fractions still equivalent to each other? Explain.

5 Use your result from part **4** in order to solve the following equation.

$$\frac{10}{7}=\frac{21}{x}$$

6 Describe how to solve proportions using this method.

7 Describe how this method relates to the method learned in Investigation 3.

There are, in fact, many ways of solving proportions. In future classes, you may encounter other methods. It is important to understand how all of these methods relate to the fundamental mathematical principles that you already know.

WEB LINK

Search for “Exploring rate, ratio and proportion” on the LearnAlberta.Ca website. In this interactive activity, you can practice equivalent 3 term ratios and watch a video and answer questions on ratios in photography.

Practice 3

1 Solve for the missing quantity in the following proportions. Round answers to the nearest hundredth where necessary.

a $1 : 4 = 2 : w$ **b** $8 : 7 = 24 : x$ **c** $p : 12 = 5 : 4$ **d** $\frac{3}{8} = \frac{m}{24}$

e $\frac{h}{57} = \frac{11}{19}$ **f** $\frac{8}{15} = \frac{19}{x}$ **g** $\frac{26}{81} = \frac{a}{100}$ **h** $\frac{31}{b} = \frac{15}{27}$

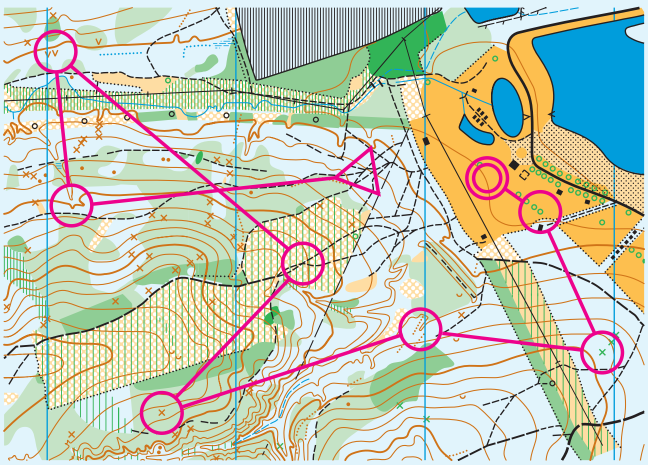
i $\frac{3}{7} = \frac{10}{y}$ **j** $\frac{12}{w} = \frac{5}{8}$ **k** $\frac{v}{21} = \frac{7}{10}$ **l** $\frac{5}{z} = \frac{45}{7}$

m $\frac{1.2}{6.8} = \frac{3.4}{c}$ **n** $\frac{4}{7.3} = \frac{f}{2.1}$ **o** $\frac{k}{43.5} = \frac{11.2}{87}$ **p** $\frac{5.5}{8} = \frac{18}{y}$

2 Determine whether each of the following pairs of ratios are equivalent. Justify your answer using the rule you discovered.

a $\frac{3}{4}$ and $\frac{42}{56}$ **b** $\frac{8}{11}$ and $\frac{21}{29}$ **c** $4:15$ and $\frac{21}{32}$ **d** $13 : 5$ and $32.5 : 13.5$

3 Orienteering competitions combine navigation with speed, where participants use a map to help navigate their route. Maps use a scale to represent distances in real-life, such as the one below.



The scale of this map is 1 : 15 000, meaning that 1 unit measured on the map is equivalent to 15 000 of the same unit measured in real life. In this example, 1 cm on the map is equivalent to 15 000 cm (or 150 m) in real life.

a If you measured 4.7 cm on the map, how far is that in real life (in meters)? Show your work using a proportion.

b How far would a distance of 350 m in real life measure on the map? Show your work using a proportion.

c On the map, the distance between two control points (circles) is 79 mm. Find the distance in real life (in meters) using a proportion.

Continued on next page

4 The Great Trail is the longest continuous recreational trail system in the world. Spanning the ten provinces and three territories of Canada, the 24 000 km trail was created through the cooperation of 477 organizations and thousands of volunteers.

- **a** The trail took 25 years to complete. Assuming a proportional relationship, find how much of the trail would have been finished after the first 10 years. Round your answer to the nearest kilometer.
- **b** After how many years would 80% of the trail have been finished? Use a proportion to find your answer and round it to the nearest year.
- **c** Assuming each organization was responsible for an equivalent amount of the trail, how many organizations would have been necessary to create the first 15 000 km of trail? Round your answer to the nearest whole number.
- **d** Canada has a population of approximately 37 million people. How many (equally sized) people would occupy one kilometer if they were to all take a position along the Great Trail? Show your working using a proportion.

5 Climbing a mountain like Mt Everest requires incredible teamwork. Not only are the conditions difficult, but the equipment necessary to safely scale the mountain can be very heavy. With a trek of about two weeks from Kathmandu to Base Camp, it is imperative that humans cooperate if everyone is to make it back safely. Groups generally hire Sherpa porters to help them with carrying the heavy equipment. If a group with 540 kg of equipment requires 12 Sherpas, how many Sherpas should be required to carry 945 kg? Show your working.

WEB LINK

For more practice with ratios and solving proportions, search for "Dunk Tank! Ratio & Proportion" on the PBS learning media website. In this interactive game, you will use ratios and proportions to solve problems in a variety of activities.

Recognizing and using proportional reasoning

As you have seen, proportions are powerful tools for solving problems. However, how do you know when a relationship is proportional? Can you solve every problem with proportions, or do only certain kinds of questions involve *proportional reasoning*?

Investigation 4 – Recognizing proportional relationships

criteria **B, C**

Many countries provide incentives for their athletes at the Olympics. While British athletes, for example, receive no monetary award for winning medals, some countries, such as India, pay their athletes for outstanding performances in order to increase their competitive spirit.

In India, the amount of money earned per gold medal is given below. This is a *proportional relationship*.

Number of gold medals	Payment (euros)
2	200 000
3	300 000
4	400 000
5	500 000

a Plot the number of gold medals on the x-axis and the payment on the y-axis. Describe the type of graph obtained.

b How much would a person be paid for winning *no* gold medals? How much would a person be paid for winning 10 gold medals? Explain how you found your answers.

In Thailand, gold medallists earn US$314 000 to be paid in equal amounts over 20 years.

c How much would an athlete be paid each year? Show your working.

d Fill in the table on the right and graph your results.

e Is this a proportional relationship? Explain.

f What do you multiply by to find the total money paid in any year? This is called the *constant of proportionality*.

g Find an equation for your graph. Explain how the constant of proportionality is represented in the equation.

Year	Total money paid (US$)
0	
1	
2	
3	
4	
5	
10	
20	

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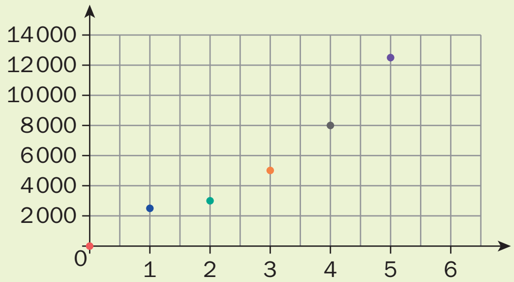
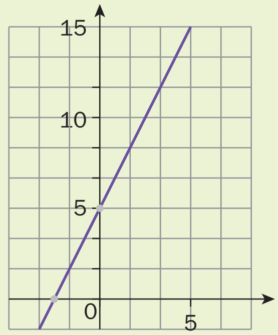
h Explain why each example below does not represent a proportional relationship.

i



ii



iii

X	*Y*
2	10
5	25
8	30
10	45
12	60

iv

X	*Y*
0	10
2	20
4	40
6	60
8	80

v

X	*Y*
1	10
2	20
3	30
4	20
5	10

i Write down a rule for determining whether a relationship is proportional, given data in a table.

j Write down a rule for determining whether a relationship is proportional, given its graph.

k Write down a rule for determining whether a relationship is proportional, given its equation.

Reflect and discuss 9

- What makes a relationship a *proportional relationship*? Explain.
- Does a proportional relationship have to include the point (0, 0)? Explain.
- Explain how the graph, equation and table all represent the same characteristics of a proportional relationship.

A *proportional relationship* can be represented using a graph, a table and an equation. Proportional relationships exist when one variable can be obtained by multiplying the other by a constant. This multiplicative factor is called the *constant of proportionality*. Once you have recognized a relationship as being a proportion, then you can use proportional reasoning to solve problems involving that relationship.

Activity 4 – Keeping the pace

A runner called a pacemaker is often used in races to force competitors to try their hardest the entire race and to promote fast times. A pacemaker is often a good runner, but is never expected to win the race. Many times, the pacemaker does not even complete it. Tom Byers was employed as a pacemaker in a 1500 m race in Oslo, Norway in 1981. His approximate time at various points during the race is given below.

Distance (m)	Elapsed time (s)
100	14.5
200	29
400	58
800	116

1 Does this seem like a proportional relationship? Explain.

2 Draw the graph of Byers' time versus the distance he covered.

3 Find an equation for your graph. Verify that it works by substituting in at least one of the points.

4 Is this a proportional relationship? Justify your answer.

5 Find Byers' time for the 1500 m race based on your findings. Show how the same result is achieved using the equation, the graph and a proportion.

6 Byers' final time was actually 219.01 seconds. Explain why this is different than the value you found.

7 Does this mean this is not a proportional relationship? Explain.

Did you know?

Steve Ovett and Steve Cram were the favorites to win the race, but they chose not to keep up with Byers' fast pace. At one point, Byers had a lead of over 70 m. Ovett, who held the world record at the time, finished the race in second place, 0.5 seconds behind Byers!

Practice 4

1 Determine whether or not each of the following is a proportional relationship. If so, find its equation.

a

X	Y
0	20
1	30
2	40
3	50

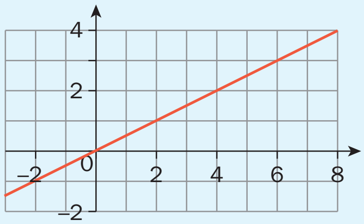
b

X	Y
3	12
5	20
7	28
15	60

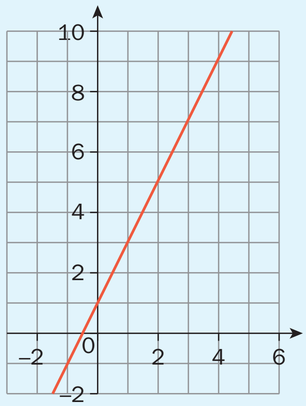
c

X	Y
0	0
1	2
2	5
3	10

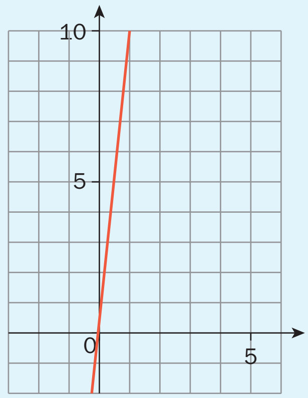
d



e



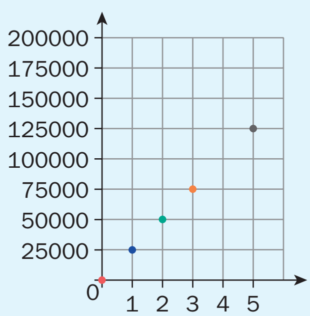
f



2 The European Economic and Social Committee (EESC) has a video and singing competition where groups of singers create a video of themselves singing a song chosen by the EESC. A group of six singers sang the required song in 4 minutes. Another group of 12 vocalists will sing the same song. How long will it take this group to sing it?

3 Rideshare companies are becoming a common mode of transportation. In order to compete with current pricing strategies, a new company called *Wheelz* is going to charge customers \$2 for every 3 minutes of driving time. How long will a 7-minute ride cost?

4 This graph represents the money earned (\$US) per gold medal at the Olympics, as promised by the United States Olympic Committee (USOC).



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a Is this relationship proportional? Explain.

b What is the constant of proportionality?

c How much money would someone who won eight gold medals receive? Show your working.

5 In 1989, two million people held hands along the side of the road through Estonia, Latvia and Lithuania. Named 'the Baltic Way', the people cooperated to put on a peaceful protest in favor of independence that achieved positive results.

Number of people	Length of chain of hands (m)
20	6.4
100	32
500	160
1200	384

a Show that this is a proportional relationship.

b Draw a graph of the data and find the equation of the line.

c Find the length in kilometers of a chain of 2 000 000 people.

6 Grocery stores often price items in bulk to seem more competitive with other stores, but they also encourage shoppers to buy more. Often, you can get the same price per item and buy fewer of them, even though the advertising might lead you to believe otherwise. The following are prices advertised by two different stores, though you are allowed to buy more or less than the amount indicated.

Healthy Granola bars:
5 boxes for $9.35

Healthy Granola bars:
3 boxes for $5.58

a Which store has the better buy? Justify your answer.

b Does buying this item from either store represent a proportional relationship? Explain.

c Find the cost of buying 15 boxes from both stores using an equation and a proportion.

d If you were to try to entice customers to buy the product from your store, give an example of how you might price it to be a better bargain, but also entice shoppers to buy more than one.

Reflect and discuss 10

ATL1

- Write down what went well in this unit in terms of your learning.
- Was there anything in this unit that you didn't understand at first, but now you do? How did you overcome that challenge?

ATL2

- You are now capable of doing your summative task. Write down a plan to complete each section before the due date.
- Create a mind map to summarize what you have learned in this unit. Compare with a peer to make sure you each have all of the concepts.
- Write down an example of how to do each kind of problem that covers a particular concept that you have learned in this unit.
- Create a daily plan to study for your unit test.

Unit summary

A ratio is a way to compare different quantities or amounts measured in the same unit.

Ratios can be written in a variety for forms, for example:

a : b	fraction	decimal	percentage
3 : 4	$\frac{3}{4}$	0.75	75%

Just like fractions, ratios can be simplified. Two ratios that can be simplified to the same ratio are said to be equivalent:

$$\frac{12}{18} = \frac{2}{3} \text{ and } \frac{6}{9} = \frac{2}{3}$$

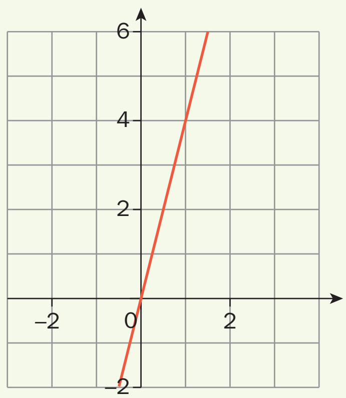
$\frac{12}{18}$ and $\frac{6}{9}$ can both be simplified to $\frac{2}{3}$. They are equivalent ratios.

Two equivalent ratios form a proportion, such as :

$$\frac{12}{18} = \frac{6}{9} \quad \text{or} \quad 12 : 18 = 6 : 9$$

A proportional relationship can be represented using a graph, a table and an equation.

Proportional relationships are straight lines that pass through the origin (0, 0).



The equation of a proportional relationship always has the form $y = kx$, where k is called the constant of proportionality.

x	y
2	6
3	9
5	15
12	36

The table of a proportional relationship can be recognized by the fact that the y-coordinate (dependent variable) is always the same multiple of the x-coordinate (independent variable). This multiplicative factor is the constant of proportionality.

Solving a proportion can be done in a variety of ways.

The products on the diagonals are equal, which produces an equation that can be solved.

$$\frac{12}{14} = \frac{18}{x} \qquad 12x = 14 \times 18$$

You can also solve it like an equation. If the variable is in the denominator, simply take the reciprocal of each ratio and then solve the equation.

$$\frac{d}{5} = \frac{6}{15} \qquad \frac{5}{d} = \frac{15}{6}$$

$$5 \times \frac{d}{5} = \frac{6}{15} \times 5 \qquad \frac{d}{5} = \frac{6}{15}$$

$$d = 2$$

Unit review

criterion A

Launch additional digital resources for this chapter

Key to Unit review question levels:

Level 1–2 | Level 3–4 | Level 5-6 | Level 7–8

1 Simplify the following ratios where possible.

a 6 : 8 b 10 : 2 c 11 : 33

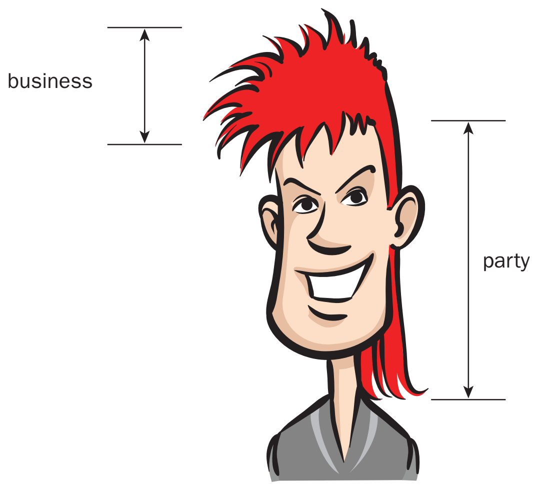
d 6 : 7 e 12 : 8 f 24 : 18

2 **Write** two equivalent ratios for each of the following ratios.

a 1 : 2 b 4 : 9 c 3 : 6

d 1 : 6 e 4 : 1 f 20 : 30

3 At the Iowa State Fair, contestants can compete in a Pigtail, Ponytail, Braid, Mullet and Mohawk competition. Competitors vie for the Blue Ribbon Award in one of the four hairstyle contests.
A mullet is a hairstyle where the front and sides are short and the back is long. Called the 'Hunnic' look in the 6th century, mullets have been described as 'business in the front, party in the back'.



Some people suggest that the ratio of the length of the hair in back (the party) to the length of the hair in front (the business) would be a good measure of 'mulletness'.

A large ratio of party : business is desired.

a **Describe** what that means in terms of which mullets are likely to win.

b Determine who wins the mullet contest given the following ratios. Show your working.

12 : 5	18 : 7	6 : 2	10 : 4	8 : 3

4 Despite heavy competition, the work force is made up of 36% college graduates and 34% high-school graduates. The remaining are neither college graduates nor high-school graduates.

a Rewrite the percentage of college graduates, high-school graduates and neither as simplified fractions.

b Find the ratio of college graduates : high-school graduates : neither.

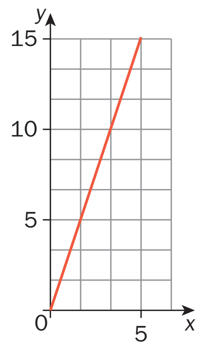
c Find the simplified ratio of college graduates : high-school graduates : neither.

5 **a** Is the relationship shown in the graph proportional? **Explain**.

b What is the constant of proportionality?

c Find the value of y when x is 24.

d Find the value of x when y is 108.



6 In 2016, the International Olympic Committee created the Refugee Olympic Team (ROT) to draw attention to the refugee crisis occurring worldwide and to honour those who 'have no home, no team, no flag, no national anthem'. In any other Olympics, the 10 athletes would have been on competing national teams, but this year they would cooperate as a unified team. The athletes and their country of origin are listed below.

Name	Country of origin
Rami Anis	Syria
Yiech Pur Biel	South Sudan
James Nyang Chiengjiek	South Sudan
Yonas Kinde	Ethiopia
Anjelina Nada Lohalith	South Sudan
Rose Nathike Lokonyen	South Sudan
Paulo Amotun Lokoro	South Sudan
Yolande Bukasa Mabika	Democratic Republic of the Congo
Yusra Mardini	Syria
Popole Misenga	Democratic Republic of the Congo

a **Write** the ratio of the number of team members from each country of origin to the total number of team members.

b Paulo Lokoro ran the 1500 m race in 4 minutes and 4 seconds, while Yiech Biel ran the 800 m race in 115 seconds. If they ran a 400 m race against each other, is it possible to tell who would win? Why or why not?

c As a refugee, Yusra Mardini had to swim and push the dinghy that she used to flee from Syria. At the Olympics she swam the 100 m freestyle event in 69 seconds. Anjelina Lohalith ran the 1500 m race in 4 minutes 47 seconds. Who had the faster speed? **Show** your working.

7 **Use** proportional reasoning to solve for the variable. Round to the nearest tenth where necessary.

a $2:9=10:p$ **b** $36:12=54:m$ **c** $11:33=y:11$

d $\frac{3}{t}=\frac{5}{11}$ **e** $\frac{8}{21}=\frac{f}{4}$ **f** $\frac{2.4}{5.7}=\frac{9.2}{k}$

8 Out of 2000 competitors at the World Iron Man triathlon, a total of 1815 completed the race.

a Assuming that male and female competitors dropped out at the same rate, find the number of male and female finishers given the ratio of male competitors : female competitors is 20 : 13.

b Find the simplified ratio of those who finished the race to those who did not.

9 A recent study found that the pay ratio of college graduates to non-college graduates over a lifetime is 184 : 100. On average, it is found that a person without a college degree will make $1.3 million ($1 300 000) in a lifetime. Given this ratio, how much would the average college graduate make in a lifetime?

10 If the ratio $x : y$ is 4 : 5 and the ratio $x : z$ is 2 : 3, then what is the ratio of $y : z$?

11 In California, there is a 'wildfire season', when wildfires are most common. These can be incredibly destructive events, with some burning as many as 1300 km^2. In 2007, the Moonlight Fire, caused by a lightning strike, affected people in three US states, requiring firefighters from across the United States to work together to get it under control. When the fire had burned 110 km^2, 1900 firefighters were battling the blaze. By the time it was under control, 2300 firefighters were putting out the fire that burned a total of 263 km^2. Because of this amazing cooperation, nobody perished and only two buildings were destroyed in the Moonlight Fire.

a Represent the ratio of firefighters to area burned at each stage of the fire in three different ways. Show your working.

b If the ratio of firefighters to area burned was maintained, how many firefighters should have been fighting the fire by the time it was under control? **Show** your working.

c **Suggest** reasons why the number of firefighters was less than the amount you calculated in part **b**.

12 At the 1996 Olympics, Michael Johnson won the 200 m race with a time of 19.32 seconds. Donovan Bailey won the 100 m race with a time of 9.84 seconds. Both men claimed to be "The World's Fastest Man".

a Who had the faster speed? **Show** your working.

The media reported that Johnson was a faster runner than Bailey.

b **Explain** why this interpretation makes sense.

c Bailey said that this is not a fair comparison. **Explain** why he is justified in saying so.

d In order to see who the 'fastest man' was, the two ran a race of 150 m in 1997. Using their results from the Olympics, **predict** the time for each in the 150 m race.

e Bailey won the race in 14.99 seconds. How does this compare to his speed for the 100 m race? **Show** your working.

13 The Human Genome Project is an international collaboration to map the three billion base pairs in the human genome, the complex set of genetic instructions. Over 1000 scientists from six different countries collaborated on the project, which took over 13 years to complete. Genome size (measured in picograms) is the amount of DNA contained in a single genome. One picogram (pg) contains 978 million base pairs.

a **Construct** a table with the number of picograms in one column and the corresponding number of base pairs (in millions) in the other. Show at least four rows.

b Is this a proportional relationship? **Justify** your answer.

c **Write** an equation for the data in your table.

d Find the number of picograms if there are 11 247 million base pairs. Show your working using both a proportion and your equation.

Summative assessment

criteria C, D

What is fair competition?

The Olympic Games, both summer and winter, are the world's leading sports competitions with more than 200 countries participating. All Olympic sports are organized by gender to insure fairness; men and women do not compete against each other. Some events, such as weightlifting and boxing, are further organized by weight, because of the unfair advantage larger athletes might have over smaller athletes. But what about height? Does the height of an athlete play a major role in the athlete's overall performance? Should some competitions also be organized by athletes' height?

Reflect and discuss 11 (as a class)

- Think of three sports in which taller athletes have a distinct advantage. Where do shorter athletes have an advantage?
- There is speculation that taller competitors have an advantage in short, fast-paced events. Do you think this is true? Give an example or two to justify your answer.

The Proportional Olympic Games

1 Track event

a Your teacher will assign you a short (less than or equal to 400 m) individual track event to analyse.

b Research who won gold, silver and bronze at the last summer Olympics in that event for both male and female competitors. In a table like the one here, record the name and height of each athlete as well as the time it took to run the race.

Medal	Female athlete	Height (m)	Time (s)	Male athlete	Height (m)	Time (s)
Gold						
Silver						
Bronze						

Suppose sprinters in this short race ran distances proportional to their height. Would that make a difference to who would have won the event?

c Find the average speed of each runner (ratio of distance : time) and represent it as a decimal. Show all of your working.

d Suppose the gold medallist runs the original distance of the event and the other medallists run a race that is proportional to their own height. Calculate the distance the silver and bronze medallists would have to run. Show all of your working.

e Assuming runners run at a constant speed, how long would it take for the silver and bronze medallists to run their proportional race? Show all of your working.

f Based on your results, who would have won the gold, silver and bronze medals in the proportional competition?

g Are the new race times closer than they were before? Did this occur in both male and corresponding female events? Does height seem to matter more with women or with men?

h Repeat the same process, assuming the silver medallist runs the original event and the other two competitors run an event that is proportional to their height.

i Repeat the same process assuming the bronze medallist runs the original event and the other two competitors run an event that is proportional to their height. Organize all of your results in a table.

j If you do make running distance proportional to height, could men and woman compete in the same event? Justify your choice.

k Based on your results, is height an unfair advantage in the track competition you selected? Justify your response.

2 **Research your own event**

a What other individual event in either the summer or winter Olympics do you think might provide an unfair advantage to taller athletes?

b Repeat the same process you did with the track event to determine whether or not height seems to matter. Organize your results in a table and be sure to show all of your working.

c On the basis of your results, is height an unfair advantage in the event you selected?

d Given your height, select one event from the two you have studied and calculate the proportional distance you would have to run/skate/cycle etc. against the gold medalist. Does that seem achievable?

Reflect and discuss 12

- How precise do your calculations have to be? To how many decimal places should you round? Explain.
- Should athletes compete in height classes for select events in the Olympics? Justify why or why not.
- Do you think you can make the decision based on the information you have? If you were to do this analysis again, is there any other information you would include?
- What makes for fair and healthy competition? Explain.
- Which is more about being equal, competition or cooperation? Explain.