MYP Mathematics

A concept-based approach

ENHANCED ONLINE

Year 3

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3 Linear relationships

Modeling with linear functions can help you understand the interconnectedness of life on Earth and the effects of human activity on this planet. However, it could also help study a wide range of topics, from sports competitions to the future of society.

Identities and relationships

Teams and competition

What does it take to win the 100 m race at the Olympics? How fast are sprinters? Is their speed constant or does it change over the course of the race? Representing this information graphically can help to analyse the contest, and predict future times in the same event.

Other competitions, such as the long jump, rowing, skiing and even archery all have elements that can be graphed, analysed and predicted. The study of linear relationships could become a study of some of the greatest races of all time or help you train in your quest to become the next great athlete.

Imagining a hopeful future

What do the gray wolf, the American alligator, the peregrine falcon, the snow leopard and the whooping crane all have in common? They have all come back from the brink of extinction. In the past, human activity has endangered many species, but there is hope that we can reverse the trend and make up for our mistakes. Linear relationships can be useful tools in studying these patterns of decline and recovery.

At the same time, there have been some positive trends in our daily lives that seem to indicate a brighter future. The wage gap between men and women is decreasing, medical breakthroughs mean that deaths from cholera and cancer are dramatically decreasing, and rights are increasingly being recognized for all humans, regardless of gender, sexual orientation, race and religion. Analysing and describing these trends with linear relationships could allow you to imagine a fairer future for all inhabitants of the planet.

3 Linear relationships

Impact of human decision-making

KEY CONCEPT: RELATIONSHIPS

Related concepts: Change, Models, Representation

Global context:

In this unit, you will explore how human behaviour affects all aspects of life on our planet. As you extend your study of **globalization and sustainability**, you will see that analyzing data and modeling relationships can help you determine trends. These patterns can then help you to understand the interconnectedness of our planet and how our decisions and actions impact others and the environment.

Statement of Inquiry

Representing patterns of change as relationships can help determine the impact of human decision-making on the environment.

Objectives

- Representing linear relationships in different ways
- Determining the characteristics of a linear relationship (gradient, y-intercept)
- Graphing linear relationships using a variety of methods
- Understanding the relationship between parallel and perpendicular lines
- Applying mathematical strategies to solve problems using a linear model

Inquiry questions

F What is a pattern?

How do you know when something is changing?

C How can you represent changing relationships?

What makes a good representation?

D How does human decision-making affect the environment?

How are we held accountable for our decisions?

ATL1 Thinking: Critical-thinking skills

Identify obstacles and challenges

ATL2 Research: Media literacy skills

Locate, organize, analyze, evaluate, synthesize and ethically use information from a variety of sources and media (including digital social media and online networks)

You should already know how to:

1 Draw a graph with a restricted domain

Draw the following graphs. Be sure to restrict your diagram to the given values of x.

$y = 2x,\ 2 \le x \le 8 \qquad y = x + 3,\ 4 \le x \le 10$

2 Identify the properties of parallelograms and squares

What are the properties of parallelograms? What are the properties of squares?

3 Substitute values into an expression.

If $a = 4$ and $b = -2$, find the value of the following:

$3a + 4b \qquad -a + 5b \qquad -3a - 6b$

4 Solve equations (including square root, fractions, squares)

Solve the following equations.

$6x - 7 = 11 \qquad 2x^2 = 8$

$\sqrt{x} - 4 = 1 \qquad \dfrac{x-5}{3} = \dfrac{1}{2}$

5 Isolate a variable

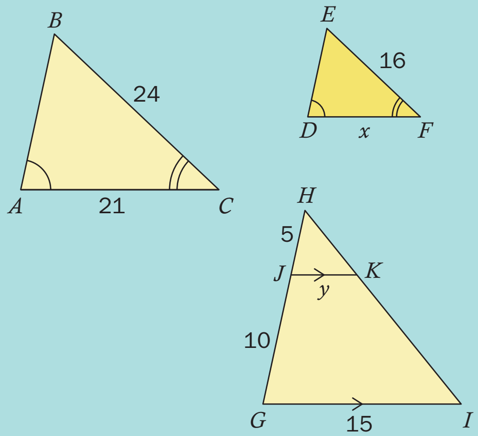
Isolate y in each of the following.

$6x - 5y = 11 \qquad 2(y - 5) = 8x$

$y - 4 = \dfrac{1}{2}(x + 3)$

6 Use properties of similar triangles to solve problems

Identify the triangles that are similar. Find the side lengths marked x and y.



Introducing linear relationships

The Greek philosopher Heraclitus once said that the only constant thing in life is change. What was he talking about? What does it mean to be constant? Can change occur at a constant rate and, if so, does that mean that the future can be predicted?

The type of relationship where the rate of change is constant is called a *linear relationship*. Any change in one variable will always produce a corresponding, predictable change in the other variable. Linear relationships are found all over the world, from the eruptions of a volcano to the build-up of greenhouse gases due to human activity and the foods we consume.

A cow releases 400 liters of methane (a greenhouse gas) a day, mainly by burping. This is the largest amount released by all the livestock animals we raise on farms.

Representing linear relationships both graphically and algebraically will allow you to model real-life situations and discover and reflect on the opportunities and tensions that result from worldwide interconnectedness. By the end of this unit, you will be in a better position to assess the impact of decision-making on humankind and the environment and to take action so that some patterns can be changed before it is too late.

Reflect and discuss 1

- What does it mean that the only constant thing in the world is change? Give an example to support your answer.
- Where do you see a change occurring at a constant rate in your life?
- Given an example of how the actions of others affect you directly. Consider actions by people on a local or even on a national level.
- How are you held accountable for your decisions?

Linear relationships

When a relationship exists between two variables, a change in one is often followed by a change in the other. You have seen in a previous course that the graph of a relationship with a constant rate of change is a straight line. This is called a *linear relationship*. What are the characteristics of linear relationships and what are the different ways that they can be represented?

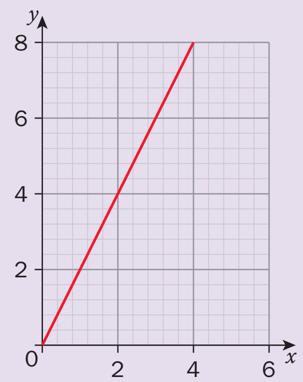
Representing linear relationships

Linear relationships can be represented in different ways, some of which are explored in the next activity.

Activity 1 – Multiple representations

1 Look at the following representations. Write down those that represent the same relationship. Justify your reasoning.

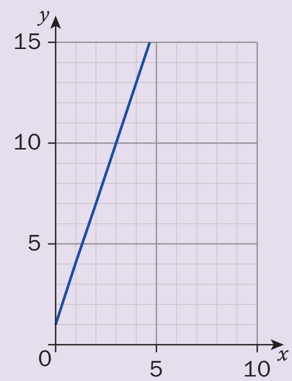
a



b (0, 1)
(1, 4)
(2, 7)
(3, 10)

c (0, 0)
(1, 2)
(2, 4)
(3, 6)

d



e “Though it had 10% battery life left, Brice’s phone was losing $\frac{1}{2}$% every hour.”

f “The temperature was 0 °C and is increasing by 2 °C every hour.”

g

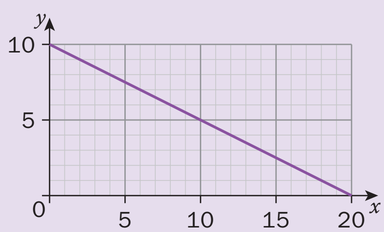
x	0	2	4	6
y	10	9	8	7

h

x	0	1	2	3	4
y	1	4	7	10	13

i “Starting with one bacterium, the number of bacteria is increasing by 3 every minute.”

j



k (0, 10)
(4, 8)
(6, 7)
(8, 6)

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2 Describe the different ways that linear relationships can be represented.

3 Describe how each representation demonstrates the same linear relationship.

4 Why is “linear” an appropriate name for this type of relationship?

Each representation has its advantages and disadvantages and your choice of which one to use will depend on the information you are analyzing and what you are trying to achieve.

Activity 2 – How much water does a leaking faucet waste?

A leaking faucet may not seem like a huge waste of water, but it certainly can add up if you leave it unfixed for a period of time. On average, a leaking faucet drips 15 milliliters of water per minute.

1 Represent this pattern with a table of values. Create a table like the one below, using 1-minute intervals for the x values up to a maximum of 10 minutes.

x (minutes)	0	1	2	3	4	5
y (milliliters)	0	15				

2 Represent the pattern as a set of coordinates.

3 Represent the pattern as a graph for these values of x and y:
$0 \le x \le 10$, $0 \le y \le 200$ (Please use a ruler when connecting the points.)

4 Represent the pattern as a verbal description of the relationship between the amount of water wasted (in millilitres) and the time (measured in intervals of 10 minutes).

5 Show that each representation will give the same amount of wasted water after 1 hour.

6 Set up a table showing the same linear relationship but using weeks as the time interval. How much water would be wasted in the course of a year?

7 Most people use about 110 liters of water in a bath. How many baths could be filled using the water from a faucet that has been leaking for one year? Show your working.

Reflect and discuss 2

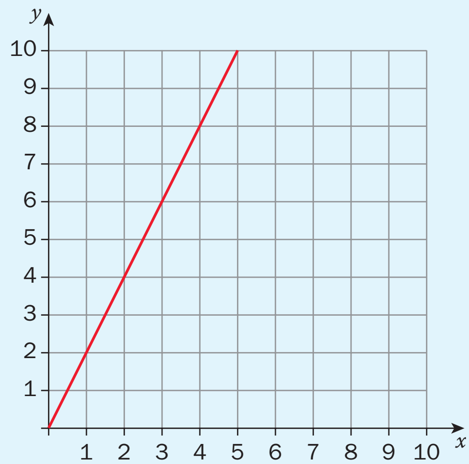
- What does a linear pattern look like in each representation (verbal description, table of values, coordinates, graph)?
- Name one advantage and one disadvantage of each representation.
- Which representation do you prefer? Explain.

In Activity 2, you were given a verbal description of a constant increase and, from the information given, you were able to generate a table of values, a set of coordinates and a straight-line graph. These are all important representations of a linear relationship. It is essential to not only recognize linear relationships, but to be able to move between these different representations.

Practice 1

1 Match each graph to the appropriate verbal description. Then state the appropriate label (including units) for the axes in each question.

a



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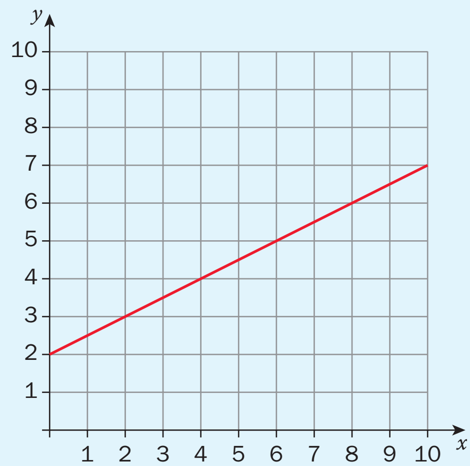
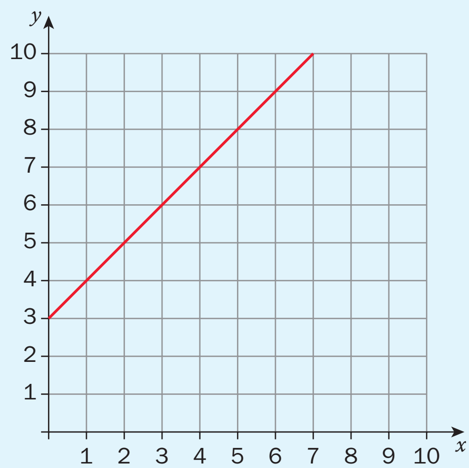


c



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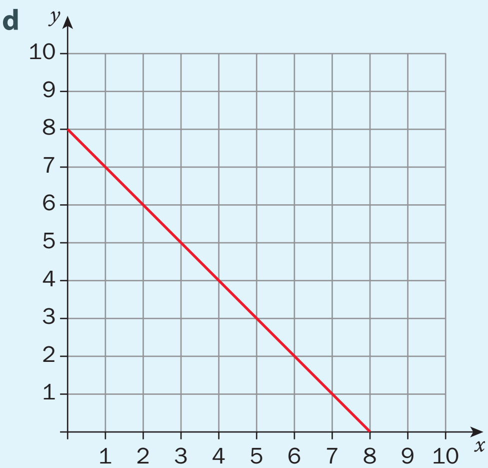

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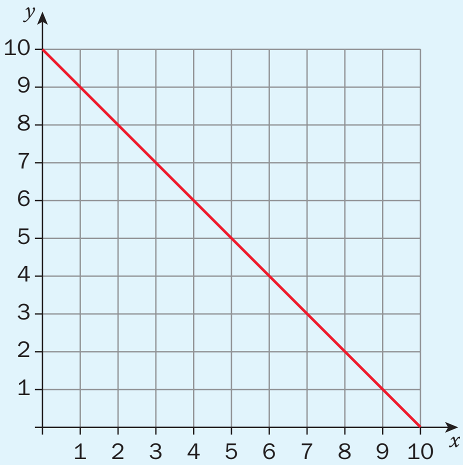
e



i	After new regulations, the level of lead in the soil, which was 10 parts per million, fell by 1 part per million every year.
ii	When replanting a deforested area from seed, the new plants grow 2 cm each month.
iii	To avoid taking a car, you hire a pedi-cab that has an initial cost of \$3 and then you are charged \$1 per hour after that.
iv	You take part in a walk-a-thon for Earth Day and are pledged €2 plus €0.50 per km walked.
v	In an effort to reduce waste, you decide to reduce, reuse, recycle and compost. The mass of your garbage one week was 8 kg but it has been decreasing by 1 kg every week because of your new habits.

2 For each of the following tables of values:

a represent the pattern as a set of coordinates

b represent the pattern as a graph with axes labelled correctly

c describe a situation that could be represented by the linear pattern.

See question 1 for examples of situations.

x	y
0	9
1	11
2	13
3	15

x	y
–6	–3
–3	–2
0	–1
3	0

x	y
–5	7
0	2
6	–4
10	–8

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3 The increase in demand for cars that run on electricity or solar power has begun to reduce our emission of greenhouse gases. Electric cars run on lithium-ion batteries, the cost of which has been dropping so dramatically that they are becoming more affordable to more people. In 2010, the cost of a lithium-ion battery was \$1000/kWh. (A kilowatt-hour, or kWh, is a unit of energy. A 1000-watt appliance used for 1 hour uses 1kWh of energy.) The price of these batteries has been decreasing by \$125/kWh every year.

a Represent this pattern with a table of values from 2010 until 2017.

b Represent the pattern as a set of coordinates.

c Represent the pattern as a graph. State your domain and justify your boundaries.

The domain is the set of all x-values.

d Represent the pattern as a verbal description of the linear relationship.

e What do you think will eventually happen to the price of lithium-ion batteries? Will the pattern continue forever? Explain your thinking.

4 Think of a real life situation that can be expressed as a linear relationship.

a Represent this pattern with a table of values.

b Represent the pattern as a set of coordinates

c Represent the pattern as a graph. State your domain and justify your boundaries.

d Represent the pattern as a verbal description of the linear relationship.

e Make a prediction based on your relationship and show that each representation will give the same result for your given domain.

5 A daily decision that most people have to make is what to do with food that they do not eat. Approximately 670 million tonnes of food are wasted each year in high-income countries and 630 million tonnes in low-income countries.

a Set up a table of values to represent the total amount of food wasted over the next 5 years for both high- and low-income countries.

b Between now and the year 2030, how many tonnes of food would be wasted in total in each type of country if this trend continues?

c How does your representation of the linear relationship help you to find the answer to question 5b?

d Is there anything about the question or your answers that surprises you? Explain.

ATL1 **e** What are the obstacles and challenges preventing people from **not** wasting food?

Characteristics of linear relationships

Linear relationships have very specific characteristics which help define them. One of them relates to the steepness of the graph while others are specific points on the graph.

Rate of change

An important property of a line is its *rate of change*. The definition of a linear relationship is that its rate of change is constant.

Constant rate of change (linear relationship)

Time (years)	Ozone (DU)
0	100
1	104
2	108
3	112
4	116

Year	Air quality ($\mu g/m^3$)
1990	120
1995	111
2000	102
2005	93
2010	84

Population (millions)	Area of forest cut (km^2)
10	1100
11	1500
13	2300
15	3100
19	4700
24	6700

NOT constant rate of change (non-linear relationship)

Time (years)	Population (billions)
0	7
1	8
2	10
3	13
4	17

Population (millions)	e-waste (millions of tonnes)
65	5.2
70	6.1
75	6.5
80	7.2
85	7.4

Year	Population (billions)
1804	1
1927	2
1959	3
1974	4
1987	5
1999	6
2011	7

Reflect and discuss 3

- Based on the above examples, explain what it means to have a constant rate of change.
- In which example was the rate of change less obvious? How were you able you tell that the rate was constant?
- How does having a constant rate of change allow you to make predictions? Explain with an example.

The rate of change is often referred to as the slope or gradient of the line. It is a measure of the steepness of the line.

If linear relationships have a constant rate of change, how do you calculate it? What does it mean?

Investigation 1 – Gradient of a line

criterion B

Incandescent bulbs were the standard light bulb for almost 150 years. However, they are very inefficient and use much more electricity than a light emitting diode (LED) bulb. You can choose to replace your regular 60 W bulb with a 10 W LED bulb, which uses one-sixth of the power and will also last 20 times longer! This will not only save you money, but it will be more beneficial to the planet.

1 An LED bulb uses 10 joules of energy every second that it is on. Represent the relationship between the number of joules used by the bulb for the first minute that it is on using a table and a graph.

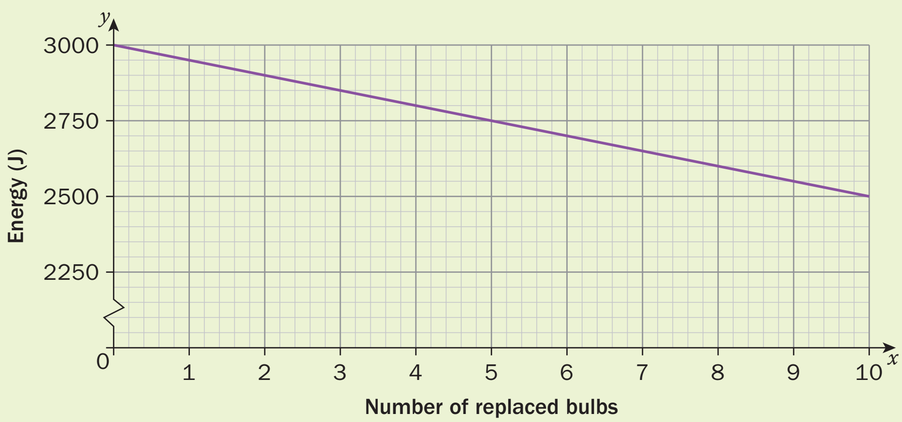
2 Choose any two points on the line and make a triangle showing the horizontal and vertical distances. The slope is the change in the vertical distance divided by the change in the horizontal distance and is defined as $\frac{\text{change in } y}{\text{change in } x}$. Determine the slope of the line you drew in step 1.

3 Choose two other points on the line and find the slope. What do you notice? Does that make sense? Explain.

4 Choose a third set of points on the line and verify your conclusion in step 3.

By changing from incandescent bulbs to LED bulbs, Maryam's electricity usage decreased, as shown in the graph below.

5 Choose three pairs of points on the line and calculate the gradient.



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6 How was the slope of this line different than the slope of the previous one? Does this make sense? Explain.

7 Based on your work, write down a formula to calculate the gradient of a line when you know two points on it. Use variables to represent the coordinates of the points.

8 Verify your rule for a new pair of points on each graph in this investigation.

9 Justify why your formula works.

Reflect and discuss 4

- What do you think is the gradient of a horizontal line? Explain.
- Why is the slope of a line $\frac{\text{change in } y}{\text{change in } x}$ rather than $\frac{\text{change in } x}{\text{change in } y}$? What difficulties would you encounter if you used the second definition instead?
- The gradient is sometimes said to be $\frac{\text{rise}}{\text{run}}$. Does this make sense? Explain.
- Which line is steeper, a line with a gradient of 5 or a line with a gradient of −5? Explain.

Parallel and perpendicular lines

Two or more linear relationships together are referred to as a *system of linear equations*. This will be the focus of Unit 7. However, two specific cases that are important here are lines that are parallel to each other and lines that are perpendicular to each another.

Reflect and discuss 5

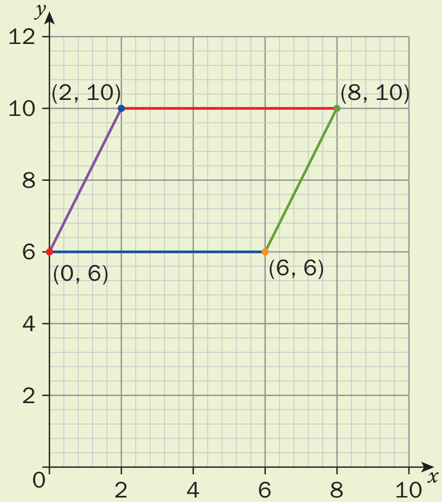
Pairs

- In pairs, discuss the meaning of parallel lines and the meaning of perpendicular lines.
- How do you think the gradients of parallel lines compare? Explain.
- How do you think the gradients of perpendicular lines compare? Explain.

Investigation 2 – Parallel and perpendicular lines

criterion B

The following diagram shows a parallelogram.

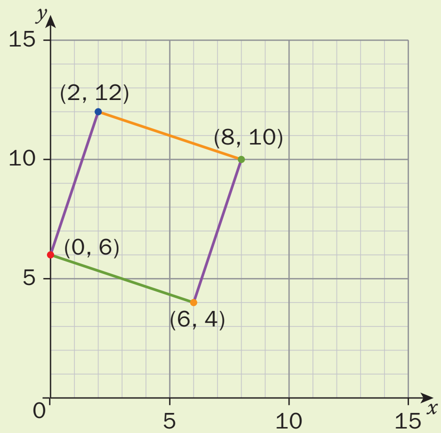


1 What are the properties of a parallelogram?

2 Prove that the diagram is actually a parallelogram by calculating the slopes of its sides.

3 Write a generalization about the gradients of parallel lines.

The following diagram shows a square.



4 What are the properties of a square?

5 You know that the angles in a square are all right angles. Use this to help you determine how the slopes of perpendicular lines compare.

6 Write a generalization about the gradients of perpendicular lines.

7 Verify your generalizations about parallel and perpendicular lines using one more example of each.

8 Justify why your generalizations make sense.

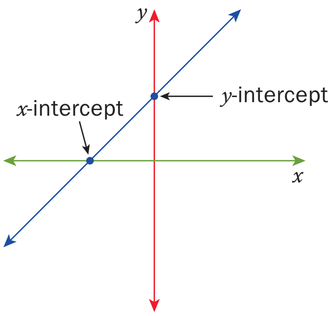
Reflect and discuss 6

Pairs

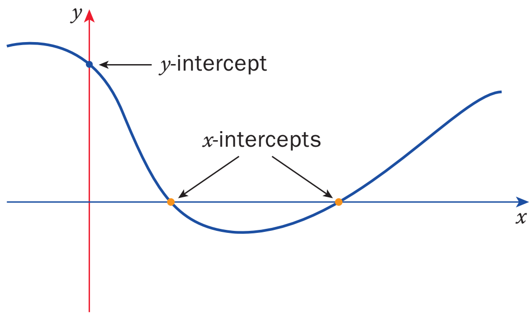
- Were your initial conjectures about the slopes of parallel and perpendicular lines correct?
- Given that the gradient of a line is a measure of its steepness, explain why what you found out about the gradients of parallel lines makes sense.
- Danika says: "The product of the gradients of two perpendicular lines is always −1." Is she correct? Explain.

Intercepts

Other important characteristics of a linear relationship are its intercepts. These are the points where the line passes through or touches the x-axis (x-*intercept*) and the y-axis (y-*intercept*). Graphs of linear relationships are not the only types of graphs that have x- and y-intercepts. Any graph that passes through or touches the x- and y-axes has these intercepts.



Linear



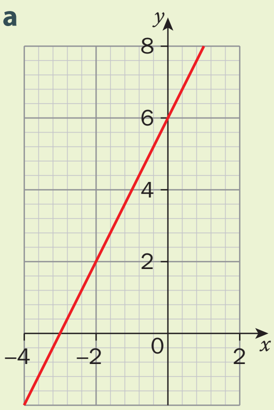
Non-linear

Investigation 3 – Finding intercepts

criterion B

1 For each of the graphs below, write down the coordinates of the x-intercept and the y-intercept. Summarize your results in a table.

a



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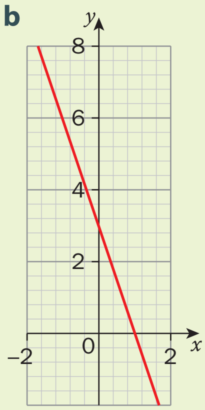
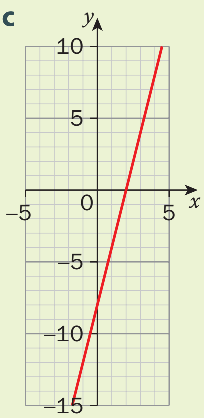
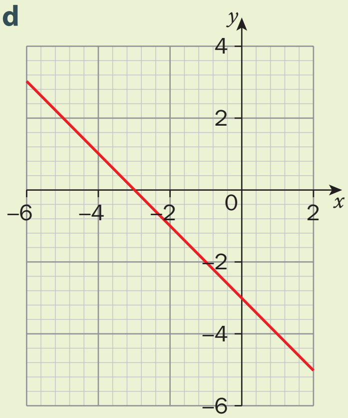


c



d



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2 What characteristic do all x-intercepts have in common? What characteristic do all y-intercepts have in common?

3 How can you use your result from step 2 to find the x- and y-intercept in the following equation?

$$2x + 3y = 6$$

4 Show that a relationship defined by $x^2 + y^2 = 4$ has x-intercepts at (0, 2) and (0, –2).

5 A relationship is defined by $x^2 + y^2 = 4$. Find its y-intercepts. Verify your answer by using an online graphing tool like Desmos.

6 Generalize a method for finding the x- and y-intercepts of a relationship given its graph.

7 Generalize a method for finding the x- and y-intercepts of a relationship given its equation.

8 Verify your method by finding the x- and y-intercepts for the relationship defined by $4x - 2y^3 = -16$.

9 Justify why your method works.

In linear relationships, the y-intercept is often named the *initial condition*. For the leaking faucet activity (see page 106), the initial condition was 0, as no water has been wasted when no time has passed. For Maryam's electricity usage (see pages 111–112), when she has replaced zero bulbs with LED bulbs, she is using 3000 J of energy per day.

Activity 3 – Saving water

Installing rainwater tanks near a house is a popular decision in countries like Australia where water shortages can occur. The water from these tanks can be used for watering gardens and even some household applications such as flushing the toilet and washing the laundry if plumbing is installed.

If a small rainwater tank holds 2000 L and is currently full, how many loads of laundry can be done if an average load uses 50 L of water?

x (number of loads)	0					
y (liters remaining in tank)	2000					

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1 Represent this pattern with a table of values.

2 Represent the pattern as a graph: $0 \le x \le 50$, $0 \le y \le 2000$.
Plot the points and then extend the line on your graph using a ruler.

3 Calculate the rate of change and state the initial condition (y-intercept) in this scenario.

4 Write down the initial condition using coordinates.

5 How many loads can be washed before the tank is empty?
Show your working.

6 Approximate the number of loads of laundry your household does in a week. How many weeks would the rainwater from a full tank last in your household?

ATL1

7 Identify one obstacle and one challenge to installing a rainwater tank in your home or school.

Practice 2

1 Indicate whether the relationship represented in each table is linear or non-linear.
For those that are linear, find the rate of change.

a

x	y
12	32
14	37
16	42
18	47
20	52

b

x	y
9	78
10	72
12	60
16	36
22	0

c

x	y
2	33
5	36
8	24
11	48
14	60

d

x	y
–2	0
1	9
4	18
8	30
13	45

2 Find the slope of each line $\overleftrightarrow{AB}$.

a $A(-3, -8)$ and $B(12, 10)$

b $A(10, 24)$ and $B(-5, 18)$

c $A(1, -20)$ and $B(\frac{1}{2}, -2)$

d $A(14, 1)$ and $B(-7, 11)$

e $A(-7, 5)$ and $B(3, 5)$

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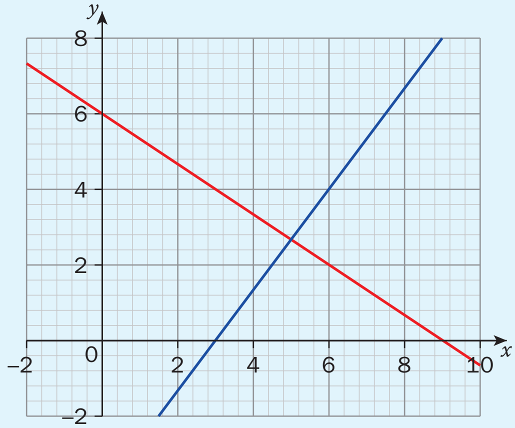
3 Are the lines shown on the grid below perpendicular? Justify your answer.



4 You want to draw a square on the Cartesian coordinate plane and you are given the coordinates $A(-5,1)$, $B(-4,4)$, $C(-1, 3)$.

a Draw these three points on a Cartesian grid.

b Where should the fourth point be placed in order to make the square? Draw the point and label it D.

c Verify that you have chosen the correct point using two different methods.

5 Find the x- and y-intercepts of the following relationships. (There may be more than one.)

a $5x - 6y = 30$ **b** $y = -3x + 11$ **c** $2x^2 - 4y = 18$

d $y = \frac{-5}{7}x + 10$ **e** $3x^2 + 2y^2 = 24$ **f** $2\sqrt{x} + 6y = 8$

6 The countries of the European Union made the decision to tackle rising greenhouse gas levels by setting targets for the emission of carbon dioxide (CO_2), one of the most prevalent greenhouse gases. The target CO_2 emission levels for vehicles in 2015 depended on the weight of the vehicle and followed a linear pattern.

2015		2020	
Weight of vehicle (kg)	**CO_2 emissions (g/km)**	**Weight of vehicle (kg)**	**CO_2 emissions (g/km)**
1300	125	1500	100
1800	150	1800	

The targets for 2020 are even more ambitious, though the line is parallel to that of the 2015 targets.

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a Represent the 2015 targets on a graph.

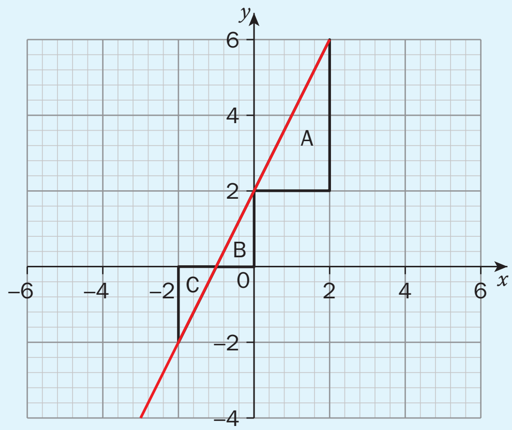
b Determine the rate of change of the linear relationship. Explain its meaning in this context.

c Using the fact that the 2020 linear relationship is parallel to that of the 2015 targets, represent the 2020 targets on the same graph as the 2015 targets to find the emissions target for 2020 for vehicles with a mass of 1800 kg.

ATL1 **d** What are the challenges of trying to set targets on a continent-wide scale?

7 In a previous unit, you determined that two triangles are similar if their angles are exactly the same size and their corresponding sides are proportional.

a Show that triangles A, B and C are similar to each other.



b Explain how these similar triangles can help you to demonstrate that a line has a constant rate of change (that the slope between any two points is always the same).

Algebraic representations of linear relationships

You have seen how you can represent a linear relationship with a graph, a table, coordinates and a verbal description. One of the most important representations is the algebraic representation, which is the focus of this section.

Recognizing linear relationships

What does the equation of a line look like? Is it possible to recognize a linear relationship without graphing it?

Investigation 4 – What is linear?

criterion B

1 Make a table with two columns, one labeled “Linear” and the other labelled“Non-linear”. You will need about 15 rows.

Linear	Non-linear

2 Using a graphing display calculator (GDC) or graphing tool like Desmos, draw the graphs of the following equations one at a time. Every time you find one that is a line, write its equation in the “Linear” column. Write the equations of the non-linear graphs in the other column.

a $y = x^2 - 5$ **b** $y = 3x + 9$ **c** $y = \sqrt{2x-8}$ **d** $y = \frac{4}{x}$

e $y = \frac{1}{2}x - 3$ **f** $y = 9 - x$ **g** $y = 2x^3 - x + 4$ **h** $y = (x+2)(x-4)$

i $y = -6x$ **j** $y = 2^x + 5$ **k** $y = \sin x$ **l** $y = 1.2x + 24$

m $y = \frac{x+3}{x-5}$ **n** $y = \frac{2x-6}{3}$ **o** $y = |x-4| + 3$ **p** $y = \frac{x}{4} - 2$

q $y = \frac{-2}{x^2}$ **r** $y = x$ **s** $y = 3(2x-12)$ **t** $y = \frac{1}{\sqrt{x+2}}$

3 Generalize your results and write a rule for how you can tell if an equation represents a straight line or not.

4 Verify your rule with two more examples.

5 Justify why your rule works.

Reflect and discuss 7

Pairs

In pairs, discuss whether or not the following equations represent linear relationships. Justify each answer.

- $4x - 2y = 10$
- $2x^2 + 3y^2 = 8$
- $\frac{2x}{3} + \frac{5y}{2} = -4$
- $y = -4x^3 + 1$

Gradient–intercept form

While all linear relationships have a constant rate of change, they can be represented in a variety of algebraic forms, such as *gradient-intercept form*.

Investigation 5 – What does this number do?

criterion B

WEB LINK

Online applications, such as the ones on the Math is Fun website or on desmos.com, will allow you to perform this investigation using sliders to alter the parameters m and c. You can use these online tools, a graphic display calculator (GDC) or graphing software.

1 Using sliders or a graphing tool, change the value of m in $y = mx$. Note the changes in the graph.

2 Using sliders or a graphing tool, select a value for m and then change the value of c in $y = mx + c$. Note the changes in the graph.

3 Using sliders or a graphing tool, change the values of both m and c in $y = mx + c$. Note the changes in the graph.

4 Generalize your results on the effects of m and c and summarize them in a table. Be sure to include the following details.

- What do m and c refer to in terms of the components of a linear relationship?
- Which parameter, m or c, affects the direction of the graph of $y = mx + c$ and in what way(s)?
- Which parameter, m or c, affects the steepness of the graph of $y = mx + c$ and in what way(s)?
- What is the effect of changing c on the graph of a linear relationship?

5 Verify your results for two more linear relationships: $y = -\frac{1}{2}x + 4$ and $y = 3x - 5$.

6 Justify why your results make sense.

Representing lines in gradient–intercept form, $y = mx + c$, has advantages when it comes to graphing these relationships. The slope and the y-intercept can become tools that allow you to draw the graph without having to use a table of values or coordinates.

Activity 4 – Graphing a line

Use the following equations in this activity.

Equation	$y = 3x$	$y = 2x - 1$	$y = -x + 4$	$y = -2x - 4$	$y = \frac{1}{2}x$	$y = -4x + 6$

1 Without using technology, make a prediction about where each of these graphs crosses the y-axis, the direction of the graph and its steepness (how many units vertically and how many units horizontally).

2 Using the information from step 1, graph each of these equations on a set of axis ranging from -10 to $+10$.

3 Verify your graphs using technology.

4 Summarize the steps you can follow in order to graph a linear relationship.

Example 1

Q For the line given by $y = 4x - 2$

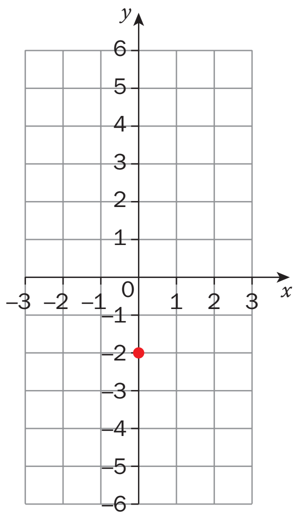
a State its slope and its y-intercept.

b Graph the linear relationship.

A **a** The slope is given by m, so this is 4.

The y-intercept is (0, –2).

b



The equation of the line is in gradient-intercept form, $y = mx + c$.

When the equation is in slope–intercept form, you start with the value of the y-intercept and plot that point on the y-axis In this case, plot the point (0, –2).

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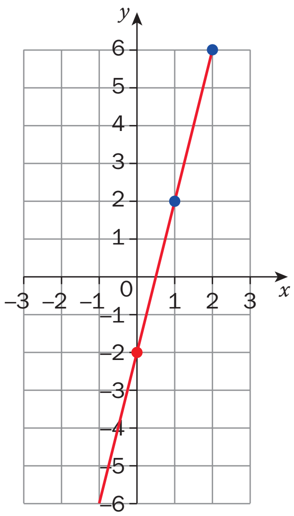
Gradient = 4

$= \frac{4}{1}$



slope = $\frac{\text{rise}}{\text{run}}$ so you convert the slope (m) into a fraction. Any whole number is represented with 1 as the denominator. If the slope is negative, put the negative sign with the numerator.

From the y-intercept you go up for the rise (or down for a negative number) and to the right for the run. In this case, you will go four units up and one to the right.

Continue this process to keep plotting points. You can also plot a point in the opposite direction from the y-intercept (in this case by going four units down and one unit to the left).
Then use a ruler to draw the straight line connecting all the plotted points.

Practice 3

1 Graph the following linear relationships using the gradient and y-intercept.

a $y = x$

b $y = -3x + 7$

c $y = 5x - 3$

d $y = -\frac{1}{2}x - 4$

e $y = \frac{2}{3}x + 1$

f $y = -4x$

g $y = 2x + \frac{1}{2}$

h $y = 3 - \frac{2}{5}x$

i $y = \frac{-x}{4}$

2 Which of the following lines are parallel? Which are perpendicular? Justify your answers.

a $y = 5x - 4$

b $y = -5x - 4$

c $y = \frac{4}{3}x + 10$

d $y = \frac{1}{5}x$

e $y = \frac{3}{4}x + 1$

f $y = -\frac{1}{5}x + 3$

g $y = -5x + 11$

h $y = -\frac{3}{4}x - 7$

i $y = \frac{-4x}{3} + 10$

Continued on next page

3 As a way of trying to limit carbon emissions in the transportation sector, train companies are changing to eco-friendly hybrid locomotives which reduce emissions by 70%. A hybrid diesel electric locomotive has an average speed of 80 km per hour.

a Copy and complete this table of values.

Time travelled in hours (x)	0	1	2	3	4
Distance travelled in kilometres (km)					

b Calculate the rate of change and the initial condition of this linear relationship.

c Represent the table of values as a graph.

d Use your graph to find how far the locomotive travels in 3.5 hours.

e Use the rate of change and initial condition to represent the linear relationship as an equation. Use your equation to verify your answer in step d.

f Explain in your own words what the rate of change and initial condition mean in the context of this situation.

4 Concrete is the world's most widely used material (apart from water) due to its versatility in construction. The drawback is that, for every tonne of concrete produced, 0.82 tonne of CO_2 is released. Increased CO_2 levels have been linked to global warming and climate change. Concrete accounts for almost 10% of the world's CO_2 emissions.

a Represent the relationship between tonnes of concrete produced and tonnes of CO_2 released for up to 100 tonnes of concrete.

b Find the rate of change and the initial condition.

c Write down the equation of this linear relationship.

d Use your equation to calculate how much concrete has to be made to release 205 000 tonnes of CO_2.

e More than 4 billion tonnes of concrete are produced each year. How much CO_2 does this release annually?

5 The technology exists to cut CO_2 emissions by up to 90% by using different concrete recipes. One developer claims it has created a concrete that can **absorb** CO_2 from the air. When water is added, every tonne of concrete absorbs 0.6 tonnes of CO_2.

a Write an equation to represent this scenario.

b If the whole world switched to this type of concrete, how much CO_2 would be absorbed by the 4 billion tonnes of concrete used each year?

c Some of these innovative concrete varieties have been available for years but have not been popular, and some of the technology has been abandoned because of lack of funds. Why do you think this is? What action needs to take place?

ATL1

d What obstacles and challenges stand in the way of this new technology being used for all concrete?

Continued on next page

6 The bottled water industry is currently worth over \$200 billion annually. The average cost of a bottle of water in the United States is 50 cents.

a Determine the equation that represents the cost (y) of buying x bottles of water. What does the gradient represent in this equation?

b What is the total cost of buying one bottle of water a day for a year (not a leap year)?

c A high-quality, refillable water bottle costs approximately \$40. How many bottles of water would you have to drink in order for this option to be more cost effective?

d What other factors, apart from cost, might inflence someone's decision whether to carry their own water bottle or to buy bottles of water?

7 The following table contains the capital cost (cost to build the plant before creating any power) and operating cost for a coal power plant and a hydroelectric power plant.

	Capital cost	Operating cost (per kWh)
Coal	\$80 000 000	\$0.20
Hydroelectricity	\$250 000 000	\$0.01

a Determine the equation that represents the cost for each type of plant. What do the gradient and y-intercept represent in these equations?

b What is the cost of producing 5000 kWh of electricity at each plant?

c What is the cost of producing 10 000 kWh of electricity at each plant?

d Coal plants need fuel, whereas hydroelectric plants do not. What will happen to the linear cost functions of both plants (gradient and y-intercept) if the price of fuel increases?

e What other factors, apart from cost, might influence a government's decision-making when considering what type of power plant to build in its country?

The total cost of electricity is the cost to build the facility plus the cost to produce electricity. You can find the cost of producing electricity by multiplying the operating cost per kWh by the number of kWh the facility generates.

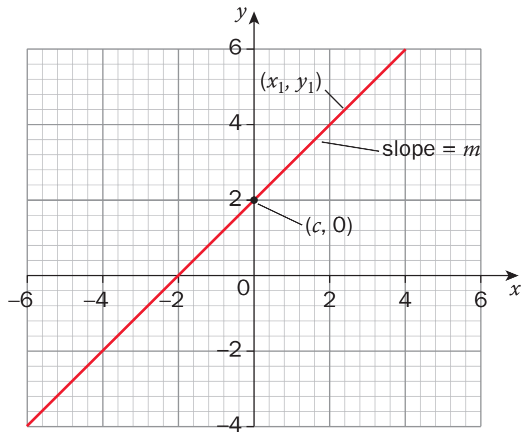
Other algebraic representations of linear equations

The equation of a line can be given in a variety of forms, one of which is gradient–intercept form. The equation of a line with slope m, y-intercept c and passing through point (x_1, y_1) can be represented in the following ways:

Slope–intercept: $y = mx + c$

Point–slope: $y - y_1 = m(x - x_1)$

Standard form: $Ax + By = C$, where A, B and C are integers.



Activity 5 – Moving between different forms

1 Copy and complete this table. Given a point and the gradient of a linear relationship, represent its equation in the three different forms.

Point and gradient	Point–slope form $y - y_1 = m(x - x_1)$	Gradient-intercept form $y = mx + c$	Standard form $Ax + By = C$
(1, 3) and $m = 2$	$y - 3 = 2(x - 1)$	$y - 3 = 2x - 2$ $y = 2x + 1$	$y = 2x + 1$ $-2x + y = 1$
(–4, –7) and $m = \frac{1}{2}$			
(2, –1) and $m = -4$			
(0, 5) and $m = -1$			
(–6, 4) and $m = \frac{2}{3}$			
(–9, 2) and $m = -7$			

2 Which form do you prefer? Explain.

3 What are some of the advantages and disadvantages of each form?

The form of the equation of a linear relationship may determine the easiest way to graph it since there are also several options for doing this.

Example 2

Q Graph the following line.

$2y + 18 = -3x$

If you are using graphing software or a GDC, the equation often needs to be in slope–intercept form.

A

$$2y + 18 = -3x$$

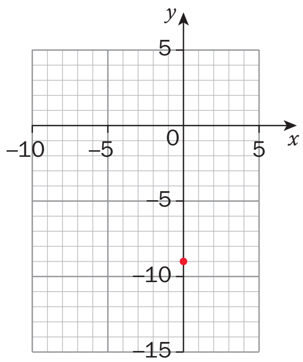
$$2y = -3x - 18$$

$$y = \frac{-3}{2}x - 9$$

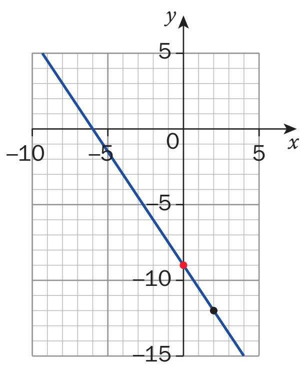
Rearrange the equation to isolate the y.

$$m = \frac{-3}{2} \text{ and } c = -9$$

From this, extract m and c.



Plot the y-intercept, which is (0, −9).



A gradient of $-\frac{3}{2}$ means that every vertical change of 3 units down corresponds to a horizontal change to the right of 2 units. From the y-intercept, go down 3 and move right 2. Join these two points and extend the line in both directions.

Are there other ways to draw a line, given the same information?

Example 3

Q Graph the following line.

$$2y + 18 = -3x$$

Two points define a line. You may find any two points, but the easiest ones tend to be the x- and y-intercepts.

A

$2(0) + 18 = -3x$

To find the x-intercept, substitute $y = 0$.

$$\begin{aligned} 2(0) + 18 &= -3x \\ 18 &= -3x \\ -6 &= x \end{aligned}$$

Solve for x.

$(-6, 0)$

Write down the ordered pair.

$2y + 18 = -3(0)$

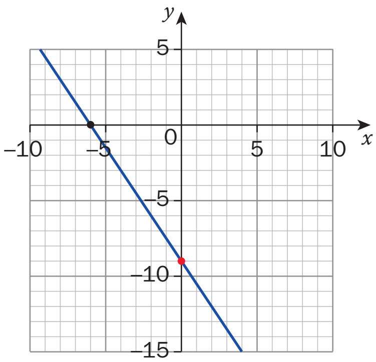
To find the y-intercept, substitute $x = 0$.

$$\begin{aligned} 2y + 18 &= 0 \\ 2y &= -18 \\ y &= -9 \end{aligned}$$

Solve for y.

$(0, -9)$

Write down the ordered pair.



Plot the two points and draw a straight line to connect them.

Reflect and discuss 8

- Is there such a thing as a **best** method of graphing a particular linear relationship? If so, can you tell from the equation what the best method might be? Explain.
- In which situation(s) is each of the three forms of linear equation most useful? Explain.
- Which method for graphing a line do you prefer? Explain.

Practice 4

1 Represent each of the following linear relationships in the form $y = mx + c$.

a $2y = -4x + 8$
b $5x - 10y = 20$
c $0 = 2x - 3y + 9$
d $\frac{1}{2}y - 3x = 5$
e $\frac{2}{3}y = 2x + 6$
f $4x = 8 - 2y$
g $\frac{3}{2}x - \frac{1}{3}y = 0$
h $\frac{x}{4} + \frac{y}{2} - 1 = 0$
i $\frac{2}{5}y - \frac{1}{10}x = -2$

2 Represent the lines in question 1 on a graph using the gradient–intercept form.

3 Find the x- and y-intercepts for each of the equations in question 1. Validate your graphs by checking to see whether these intercepts are on the lines.

4 Which of the following lines are parallel? Which are perpendicular? Justify your answers.

a $x - 7y = 10$
b $2x - y = 8$
c $3x + 6y = -2$
d $6x + 4y = -5$
e $y = \frac{1}{7}x - 5$
f $8 - 7x = -y$
g $28y = 4x - 12$
h $8x + 4y = -3$
i $y = -2x + 9$
j $7x = 18 - y$
k $4x + 2y = 1$
l $y = \frac{2}{3}x$

5 Graph the following lines by plotting the x- and y-intercepts.

a $3y + 4x = 24$
b $5x - 10y - 30 = 0$
c $0 = 2x - 4y + 12$
d $\frac{y - 3x}{2} = 6$
e $\frac{1}{4}y = 2x + 4$
f $4y = \frac{1}{2}x - 12$

6 Graph the following lines using a method of your choosing.

a $y = 3x - 12$
b $3y = 5x + 15$
c $0 = -3x + 14 - 7y$
d $y = -x + 4$
e $4y - 8x = 12$
f $\frac{y - 2}{3} = -2x$
g $2y = 2x - 10$
h $-2 = 3x - \frac{1}{2}y$
i $\frac{1}{3}x - \frac{2}{5}y = 0$

7 Using asphalt to create roads and parking lots has created "urban heat islands" because the asphalt absorbs the Sun's rays and radiates the heat back into the atmosphere. Temperatures in these zones are much hotter than in suburban and rural areas, in some cases causing severe health issues. To combat this, the city of Los Angeles elected to paint the asphalt with a special material that reflects the Sun's rays.

a Crews can paint $4\,m^2$ each minute. Represent this relationship using a table, a graph and an equation written in all three forms. Show your working.

b What do the gradient and y-intercept mean in the context of this situation?

Continued on next page

c Because of this new technology, temperatures cool off much faster at night than without the new material. Suppose the temperature at 6 pm is 28°C and it drops by 0.5°C every hour. Represent this relationship using a table, a graph and an equation written in all three forms. Show your working.

d What do the gradient and y-intercept of this new equation mean in the context of this situation?

Activity 6 – Turn it "OFF"

The World Wide Fund for Nature issues tips on how to lessen your impact on the environment.

One of these tips is to completely turn off equipment when not in use and to unplug chargers, even if no device is attached. Approximately one-quarter of all residential energy consumption in industrialized countries is used by electrical devices in "idle" power mode. These devices and chargers use what is called "standby power" or "phantom power".

1. Find out the cost per kilowatt hour that your electricity provider charges.
2. Look at your current electricity bill and assume that 25% is wasted on devices in "idle" mode.
3. Create a linear equation (or *linear model*) to represent the cost of phantom power in your home. What is your x variable? What is your y variable?
4. Describe the rate of change and the initial condition in the context of this question.
5. Represent your model as a graph.
6. Use the model to predict how much energy and money you could save in a year if you followed the given tip. Show all your calculations.
7. ATL2 Go through your house and make a list of all devices and cords that draw phantom power. Research ideas from a variety of reputable sources and create an action plan for easy ways to save energy in your home.
8. ATL1 What are the challenges involved in making the choice to turn off or unplug your computer/phone charger/television when you are not using them?

Determining the equation of a line

In order to determine the equation of the line, you simply need a point on the line (x, y) and the gradient of the line (m).

Example 4

Q Find the equation, in slope–intercept form, of a line that passes through (2, 7) and has a slope of 5.

A

$y = mx + c$

Start with the equation and substitute in the values you have been given.

$7 = (5)(2) + c$

$7 = 10 + c$

Solve for the y-intercept.

$-3 = c$

$y = 5x - 3$

Write down the equation in the required form.

Example 5

Q Find the equation, in standard form, of a line that passes through (9, –2) and (–15, 6).

A

Calculate the slope by using the formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{6 - (-2)}{-15 - 9}$$

$$m = \frac{8}{-24}$$

$$m = -\frac{1}{3}$$

Substitute the slope and the coordinates of one point into the equation.

$$y = mx + c$$

$$-2 = \left(-\frac{1}{3}\right)(9) + c$$

$$-2 = -3 + c$$

$$1 = c$$

Write down the equation in gradient–intercept form.

$$y = -\frac{1}{3}x + 1$$

Multiply both sides of the equation by 3 so that all coefficients are integers. Then write the equation in the required form.

$$3y = -x + 3$$

$$x + 3y = 3$$

Example 6

Q Find the equation of the line through (6, –8) that is perpendicular to the line $y = 3x + 5$. Write your answer in slope–intercept form.

The ∴ symbol means "therefore".

A

$$m = 3$$

$$\therefore\ m_{\text{perp}} = -\frac{1}{3}$$

Determine the slope of the linear relationship. If it is parallel to the given line, use the same slope. If it is perpendicular, use the negative reciprocal.

$$y = mx + c$$

$$-8 = \left(-\frac{1}{3}\right)(6) + c$$

$$-8 = -2 + c$$

$$-6 = c$$

Substitute the slope and coordinates of the point into the equation.

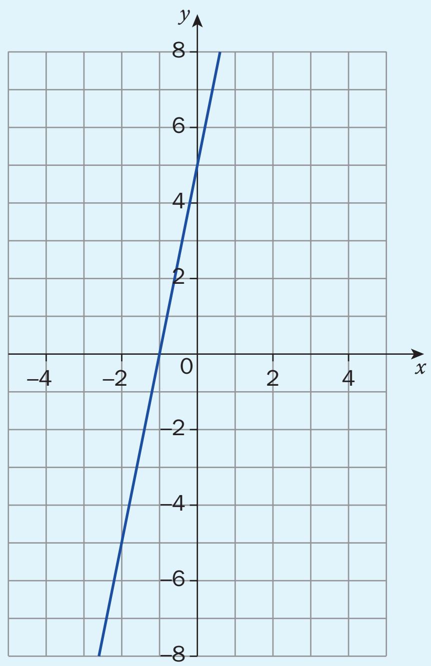
$$y = -\frac{1}{3}x - 6$$

Write down the equation in the required form.

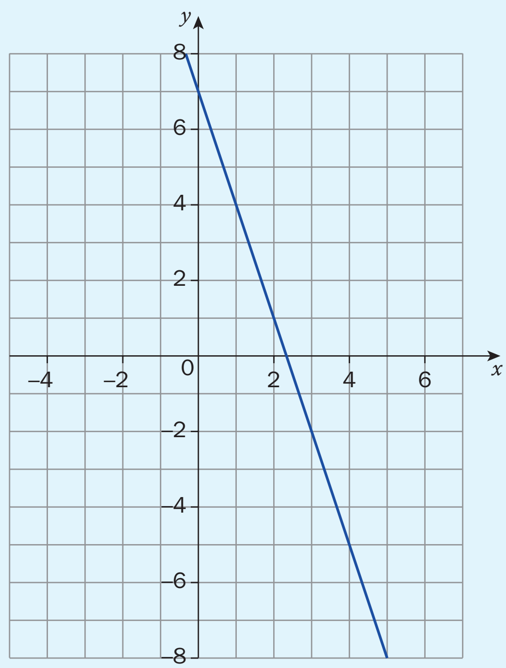
Practice 5

1 Represent the following linear relationships algebraically. Write your answers in all three forms (gradient–intercept, point–slope and standard form).

a



b



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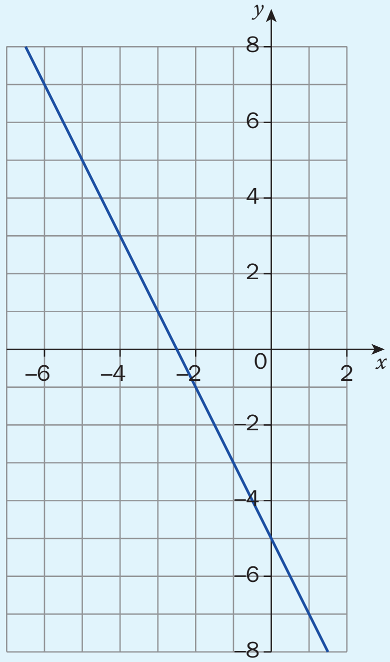
c



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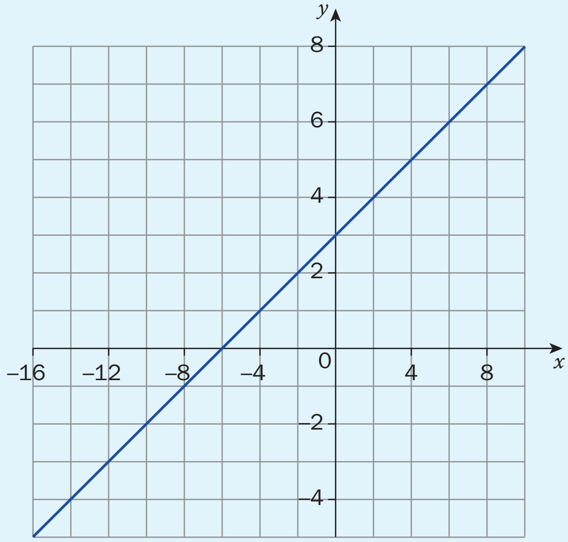


2 Find the equation of the following lines. Write your answers in gradient–intercept form without using decimals.

a The line with a slope of –4 and a y-intercept of 5

b The line with a slope of $\frac{1}{3}$ and passing through (2, 3)

c The line with a slope of –2 and an x-intercept of 5

d The line passing through (–6, –2) and (4, 3)

e The line with a y-intercept of 4 and an x-intercept of –3

f The line with an x-intercept of 7 that is perpendicular to the line $3x - 6y = 8$

g The line that passes through the origin that is perpendicular to the line $2x + 7y = 12$

h The line that passes through (–1, 1) and is perpendicular to the line that passes through (4, –2) and (–3, 1).

3 The line through (2, –4) and (x, 1) has a slope of –5. Determine the equation of the line and the value of x.

4 According to the International Union for Conservation of Nature, between 2006 and 2013 the African elephant population decreased from 550 000 to 470 000, and the number continues to decrease. Despite warnings and stiffer penalties, illegal poaching and selling elephant ivory, which are the main causes of this decline, continue.

a What is the average number of elephants that died each year between 2006 and 2013?

Continued on next page

b Assuming the population continues to decrease at this average rate every year, represent the African elephant population since 2013 with a linear model. Use your equation to predict the African elephant population this year.

Set 2006 to be year 0 and determine the equation.

c Assuming the population continues to decrease at the same rate, when will the African elephant become extinct?

5 As the Earth heats up, sea levels rise because warmer water takes up more room than colder water, a process known as thermal expansion. Melting glaciers compound the problem by dumping more fresh water into the oceans. In 1993, NASA started measuring sea levels. Since then, the sea level has risen an average of 3.2 millimetres per year.

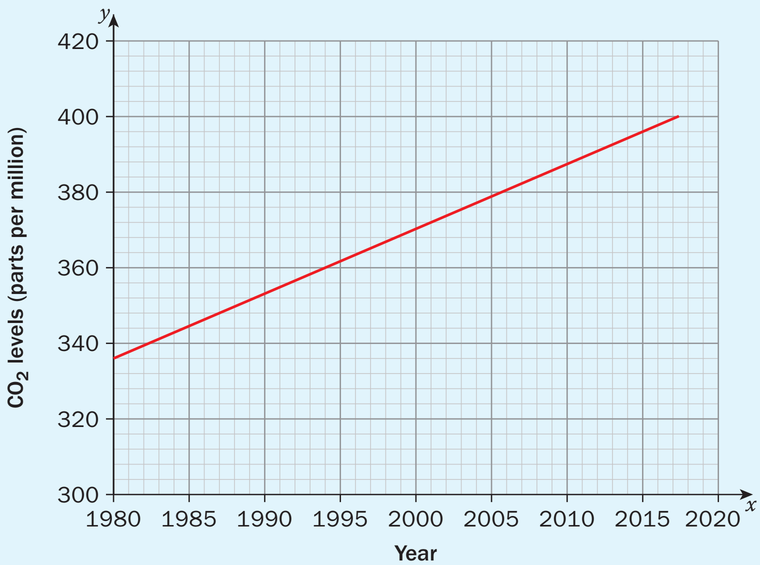
a Set up a linear equation to model this phenomenon. According to this data, how much has the sea level risen since NASA started recording the data?

b Worldwide, approximately 100 million people live within three feet (94 cm) of sea level. If the current trend continues, when will all of these people's homes be flooded?

WEBLINK

Search for the video entitled "How Earth Would Look If All The Ice Melted". You can see what could happen to major cities all over the world if the polar ice caps and permafrost on Earth melted.

6 This graph represents global carbon dioxide levels in the atmosphere since 1980.



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a According to the graph, what was the CO_2 level in 1990? What was it in 2017?

b According to the graph, what is the change per year of CO_2 levels in the atmosphere?

c Use the graph to determine the equation of the line, using 1980 as your initial condition (y-intercept).

d Verify your equation using different points on the graph. Discuss any discrepancies you may have.

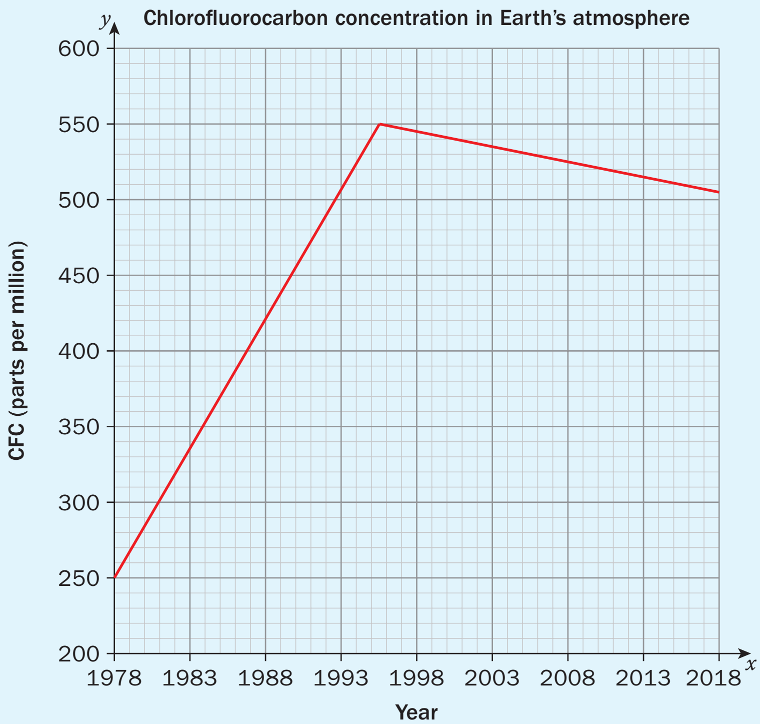
e Researchers have discovered that Antarctica's ice will be significantly more vulnerable to melting once CO_2 levels exceed 600 parts per million. When will this occur if the current trend continues?

ATL2

f Research your country's carbon dioxide emissions per person. How does your country compare with the rest of the world? Why do you think this is?

> Be sure to check a variety of reputable sources. Reference these in your answer and state how you know the source is reliable.

7 Chlorofluorocarbons (CFCs) were created in 1928 and were used in products such as aerosol propellants, cleaning solvents and refrigerants. It has since been proved that CFCs contribute to the destruction of the ozone layer. They continued to accumulate in our atmosphere until a global decision was made to address this potentially catastrophic issue.



A graph that contains two sets of trends, like this one, can be referred to as a piecewise relationship.

Continued on next page

ATL2

a Using a variety of sources, research the global effort to ban CFCs that officially started in the late 1980s. Do the sources say it was successful? Explain how you know you can trust the points of view presented.

b Use the graph to determine whether the global agreement to ban CFCs was successful. Explain your answer.

c According to the graph, what was the rate of change in CFC concentrations in the atmosphere leading up to the global ban? Verify your answer using different points on the graph. Discuss any discrepancies you may have.

d According to the graph, what was the rate of change in CFC concentrations in the atmosphere after the global effort started? Verify your answer using different points on the graph. Discuss any discrepancies you may have.

e Although much has been accomplished, CFCs decompose very slowly, so even when all CFC production has stopped (which is not yet the case), some concentration of CFCs will remain in the atmosphere for over 100 years. According to the decreasing trend, in what year will there be zero CFCs left in the atmosphere?

f Write down two facts that you have learned from this case study and its graph.

Formative assessment

criteria C, D

Reducing the emission of greenhouse gases has been a major focus of governments across the globe since the mid-1990s. In 1997, most countries signed the Kyoto Protocol, agreeing to reduce greenhouse gases and their effect on global temperatures.

The following data represent the CO_2-equivalent concentration of all greenhouse gases in the atmosphere in parts per million over a span of 20 years.

1995	427
2000	441
2005	455
2010	469
2015	483

a Plot these points on a graph, making sure you start your x-axis at 1990 (as that was the benchmark year agreed in the Kyoto Protocol). Plot the years from 1990 to 2030.

b According to your linear model, what was the concentration of the mix of greenhouse gases in 1990?

Continued on next page

c According to your linear model, what is the change per year in the concentration of greenhouse gases in the atmosphere?

d Use your graph to determine the equation of the line, using 1990 as the initial condition. Write the equation in the three different forms.

e Verify your answer to step **d** using algebra.

f According to your model, what is the concentration of greenhouse gases this year?

g If the Kyoto Protocol threshold is 500 ppm for the concentration of greenhouse gases, in what year will the concentration reach that amount if current trends continue?

h Draw this threshold on your graph as a dotted horizontal line. Continue your line to show the point at which it intersects the threshold. Verify that your graphical results are the same as your algebraic results.

Reflect and discuss 9

- What is the Kyoto Protocol, when was it adopted and when was it enforced?
- How many countries agreed to the Kyoto Protocol?
- Given this agreement, would you expect the greenhouse gas emissions data that you graphed in the Formative assessment to continue to increase in the short term? In the long term? At what rate?
- ATL1 What are some obstacles to reaching and enforcing an agreement like the Kyoto Protocol?
- Whose responsibility is reducing greenhouse gases? Who should be held accountable – industry, government or individuals? Explain.

Vertical and horizontal lines

So far, the lines you have studied have all been diagonal or sloping. What happens when the line is vertical or horizontal? Can its steepness be measured? How can you represent these lines with equations?

Investigation 6 – Equations of vertical and horizontal lines

criterion B

1 Graph the following lines by first creating a table of values for each line. Note that $y = 2$ means that the value of y is 2 for any value of x. Graph all three lines on one set of axes.

$y = 2$	
x	y

$y = 3$	
x	y

$y = -5$	
x	y

2 How are the lines related? What type of lines are they?

3 Pick any two points on each line. Calculate the slope of each line.

4 How are the slopes related? How does the slope relate to the steepness of the graph?

5 Repeat step 1 for the following lines. Note that $x = 2$ means that the value of x is 2 for any value of y.

$x = 2$	
x	y

$x = 3$	
x	y

$x = -5$	
x	y

6 How are these lines related? What type of lines are they?

7 Pick any two points on each line. Calculate the slope of each line.

8 How are the slopes related? How does the slope relate to the steepness of the graph?

9 Write a generalization about lines that are of the form $x = a$, where a is a number.

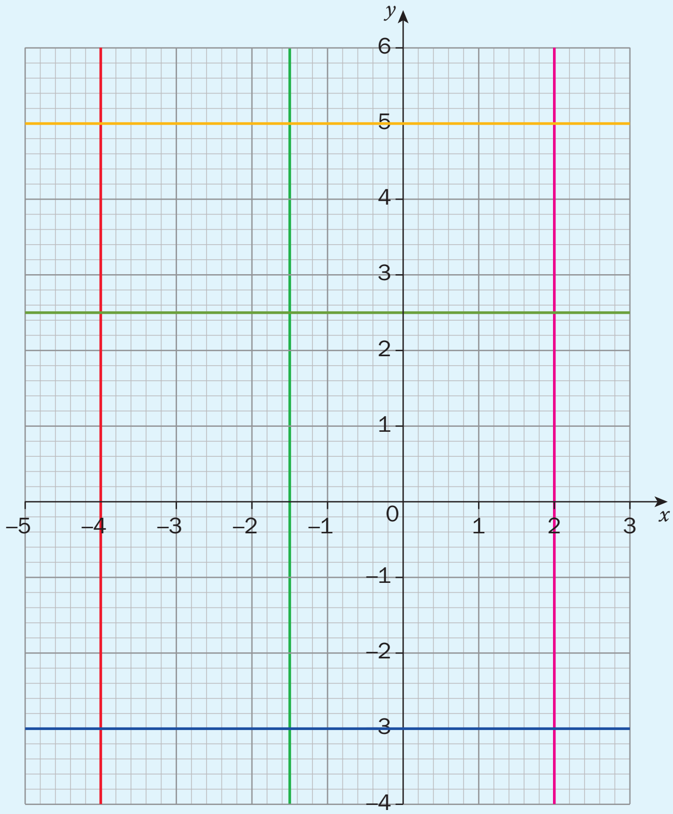
10 Write a generalization about lines that are of the form $y = b$, where b is a number.

11 Verify your generalizations for one more case of each type.

12 Justify why your generalizations are true.

Practice 6

1 Find the equations of the lines shown on the set of axes below.



2 Find the equation of the line through (6, 1) that is parallel to the line $y = 34$.

3 Find the equation of the line through (0, 1) that is perpendicular to the line $x = -2$.

4 Find the equation of the line through (5, 5) and (5, –2).

5 Find the equation of the line through (–3, –2) and (1, –2).

6 Find the equation of the line through (4, 2) that is parallel to the line $x = -8$.

7 Find the equation of the line through (–7, 8) that is perpendicular to the line $y = -4$.

Unit summary

Linear relationships can be represented in a variety of ways:

- graph
- table
- coordinates
- words
- algebraic equation.

Some of the characteristics of a linear relationship are its gradient (or slope), its x-intercept and its y-intercept.

The gradient or slope of a line is a measure of its steepness. It is defined as the ratio of its vertical change (rise) to its horizontal change (run). The formula for the gradient of the line joining the point (x_1, y_1) and (x_2, y_2) is

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

A **positive gradient** slopes upwards from left to right.
A **negative gradient** slopes downwards from left to right.

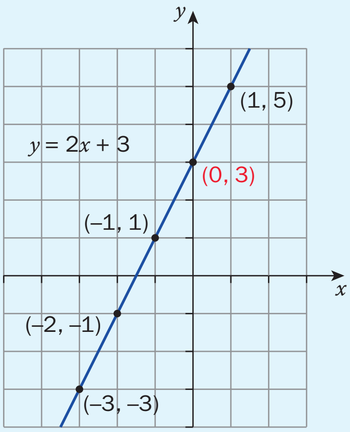
If two lines are parallel, then their gradients are the same.
If two lines are perpendicular, then their gradients are negative reciprocals of each other (for example, $\frac{2}{5}$ and $-\frac{5}{2}$). Another way to state this relationship is that the product of the slopes of perpendicular lines equals −1. For example $\frac{2}{5} x - \frac{5}{2} = -1$.

The x-intercept is the point where a graph crosses the x-axis. Given the equation of the graph, you can find the x-intercept by substituting $y = 0$ and solving for x.

The y-intercept is the point where a graph crosses the y-axis. This is also referred to as the initial condition in the context of application questions. Given the equation of the graph, you can find the y-intercept by substituting $x = 0$ and solving for y.

To graph a line:

1. Plot the y-intercept on the coordinate axes.
2. Starting from the y-intercept, use the gradient to determine the coordinates of another point on the coordinate axes. Continue this process for a few more points.
3. Draw a line between the y-intercept and the other points.



A linear relationship can be represented in one of three forms:

Gradient–intercept or (slope–intercept) form: $y = mx + c$

Point–gradient or (point–slope) form: $y - y_1 = m(x - x_1)$

Standard form: $Ax + By = C$, where A, B and C are integers.

Given the graph of a line, you can determine the equation of the line in two different ways:

- Select two points on the graph, find the gradient and then use the gradient and a point to write the equation in point gradient–form.
- Use the gradient and where it cuts the y-axis to find the y-intercept, and then write the equation in gradient–intercept form.

Horizontal lines have a gradient of zero. The equation of a horizontal line is always of the form $y = a$.

Vertical lines have a gradient that is undefined. The equation of a vertical line is always of the form $x = a$.

Unit review

criterion A

Launch additional digital resources for this chapter

Key to Unit review question levels:

Level 1–2 | Level 3–4 | Level 5–6 | Level 7–8

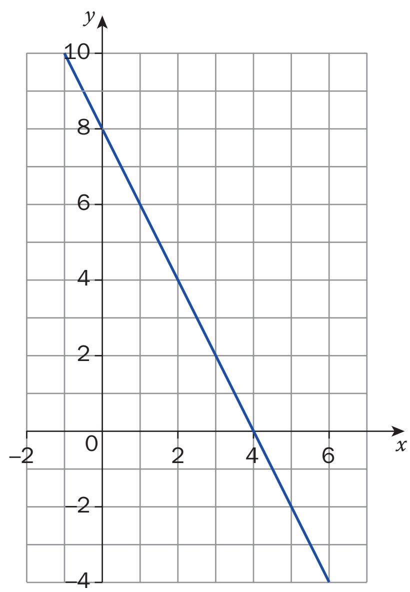
1 **State** the slope and the y-intercept of each of the following lines.

a $y = 6x - 12$

b $y = -\frac{2}{7}x + 23$

2 Determine the x- and y-intercepts of the following lines.

a



b $5x - 4y = 20$

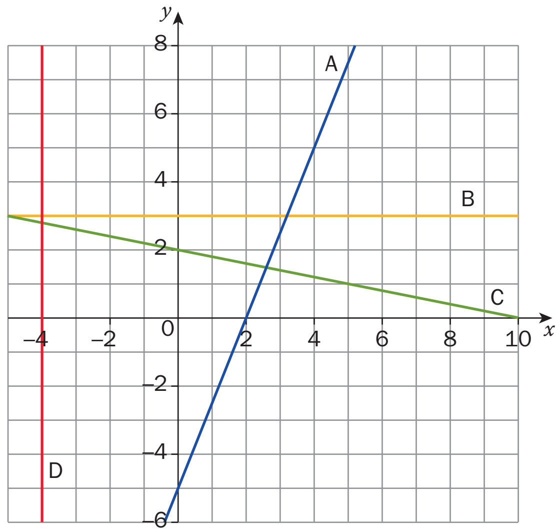
c $7x + 3y = 42$

3 **Plot** the graph of each of the following lines using a method of your choosing.

a $y = -2x + 8$

b $y = x - 7$

4 Copy and complete the table for lines A, B, C and D.



Line	Slope	y-intercept	x-intercept	Equation
A				
B				
C				
D				

5 Represent each of the following equations using gradient–intercept form.

a $3y = \frac{1}{2}x - 12$ **b** $5x = 10y - 20$ **c** $x = 8y$

d $\frac{1}{2}x = 3y - 2$ **e** $\frac{2}{3}y - 8x = -6$

6 Find the gradient of the line L that passes through each pair of points.

a $A(1, 5)$ and $B(1, -2)$ **b** $A(-1, 6)$ and $B(2, 9)$

c $A(0, 4)$ and $B(2, 8)$ **d** $A(3, 2)$ and $B(5, 4)$

7 A tank has a slow leak in it. The water level starts at 100 cm and falls 0.5 cm a day.

a Is this a constant increase or constant decrease situation?

b **Write down** an equation showing the relationship between day, d, and water level, L.

c After how many days will the tank be empty?

8. Graph each of the following lines using a method of your choosing.

a $6y + 3x = 12$ **b** $10y - x = -50$

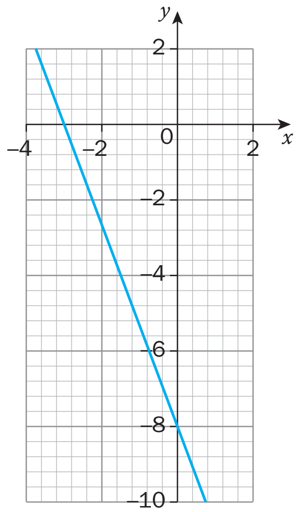
c $2y + \frac{1}{3}x = 10$ **d** $2y + 3 = \frac{x}{4}$

e $0 = 24 + 8x + 3y$ **f** $4x = \frac{10 + 2y}{3}$

9. Match each of the following equations to its graph.

a $3x - 5y = 15$ **b** $y = \frac{1}{2}x$ **c** $\frac{4}{3}x + \frac{1}{2}y + 4 = 0$

i



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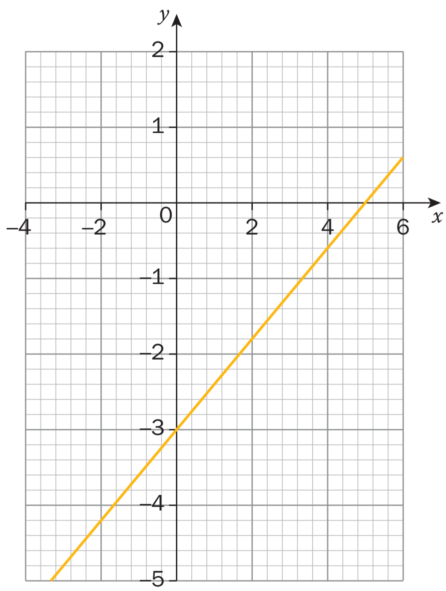

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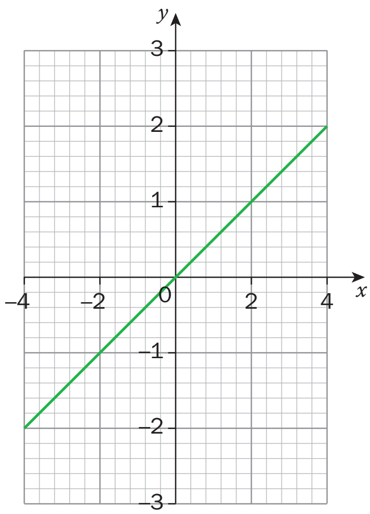


10 The line through $(2, -4)$ and $(c, 1)$ has a slope of -5. Determine the value of c.

11 Write the equation of the line parallel to $3x + y - 4 = 0$ that passes through the point $(2, -5)$.

12 Find the equation of a line that goes through $(-3, 9)$ and $(9, 1)$.

13 Write the equation of the line perpendicular to $3x + y - 4 = 0$ that passes through the point $(2, -5)$.

14 Write the equation of the line parallel to $x = -3$ passes through $(6, -7)$.

15 Determine the equation of the line perpendicular to $y = 4$ that passes through $(-1, 6)$.

16 In 2016, there were a total of 36.63 million people already living with HIV, and there were 1.8 million new HIV infections in that year. The rate of new infections is decreasing by approximately 300 000 people per year due to advancements in drug technology and the support of global agencies and local governments.

a Write a linear equation that models the number of people in millions who have been newly infected with HIV since 2016.

b If the rate of infections continues to follow this linear trend, in what year will there be no new infections?

c Is this a realistic model to use for such an epidemic? Why or why not?

17 According to the National Academy of Sciences of the United States of America, there were 7100 cheetahs left in the wild in 2016. The number had decreased significantly from an estimated 14 000 in 1975, when the last comprehensive count was done in Africa. The decision to convert wilderness areas to agricultural or livestock farms has caused the loss of habitat that is the main cause of this decline.

a What is the average decrease in the number of cheetahs each year?

b Assuming this is still the average rate of decline, what is the cheetah population this year?

c Assuming this average rate of decline continues, when will the cheetah become extinct?

Set 1975 to be year 0 and determine the equation.

18 Determine the equation of the line perpendicular to the line $4x + 2y - 7 = 0$ that has the same x-intercept as the line $2x + 3y - 12 = 0$.

19 Find the value of k if the lines $3x - 2y - 5 = 0$ and $kx - 6y + 1 = 0$ are:

a parallel

b perpendicular.

20 Triangle ABC has vertices $A(3, -1)$, $B(-3, -5)$ and $C(-1, 5)$. Determine whether the triangle is a right triangle. Fully justify your answer.

21 In 1991, 18.6% of the world population was undernourished. In 2015, 10.9% of the population was undernourished.

a What was the average percentage change per year?

b Assuming this rate continues, what is the percentage of the population that is undernourished this year?

c Determine an equation to represent this scenario.

d Assuming this rate continues, when will there be no undernourished people in the world? Do you think this is actually possible?

e What decisions at local, country and world level could be driving this change?

22 The decision of which car to buy is no longer just about color and style; it may also involve choosing one that is environmentally friendly. A cost comparison of cars is now a necessity when shopping around for a new car. In the United States, the Ford Focus sells for \$19 000 and costs 8 cents per mile to run. The zero-emission electric Focus sells for \$21 600 and costs only 3 cents per mile to run.

a Determine the equation that represents the relationship between the total cost of each car (including both buying and running costs) and the miles traveled. What do the gradient and y-intercept represent in these equations?

b What is the total cost of each car at 30 000 miles? If you were planning to sell the car once it reached 30 000 miles, which car would be the cheapest option?

c What is the total cost of each car at 60 000 miles? If you were planning to sell the car once it reached 60 000 miles, which car would be the cheapest option?

d Which characteristic of these linear relationships (gradient or y-intercept) would be affected if

i the price of gasoline in the US increases?

ii the US government offers a larger rebate (a partial refund) on the purchase of new electric cars?

e When is the total cost of each car exactly the same? Write a general statement relating the mileage of the cars to when each model is the cheapest, using your original equations.

f What other factors apart from cost might go into your decision of which car to buy?

23 On average, 3.85 kilograms of feed are needed for every kilogram of meat we consume (this is called the *feed conversion ratio*).

a Represent this information in a table showing the relationship between kg of feed and kg of animal meat. Your table must have at least 6 rows of data.

b **Sketch** this linear model, with kg of feed on the x-axis and kg of animal meat on the y-axis. Even if your data doesn't go that high, make sure your x-axis goes to 130 kg.

c Research the most recent meat consumption per capita (per person) data for your country. **Draw** a horizontal line on your graph to represent this.

d How many kilograms of feed are needed to sustain the livestock raised for one person's meat consumption in your country?

e Research the population of your country. What is the overall weight of the feed needed to meet your country's annual meat demands?

f If corn crops yield approximately 4445 kg of grain per acre annually and 60% of that is fed to livestock, how many acres need to be planted with corn in order to feed the livestock eaten in your country?

g To put this area in perspective, research the area of the city or town in which you live. How does it compare with the area of farmland needed to feed the livestock?

Did you know…?

Edible insects are a sustainable, ecologically viable meat source and could actually be a feasible replacement for some of the meat we eat. Crickets, for example, have a feed conversion ratio of 1 to 1 so they are more efficient at producing meat than any of the typical animals we eat. The average edible insect is also approximately 50% protein, compared with the lean cuts of the big four meats which range from 28% to 34%.

ATL2

Summative assessment

criteria C, D

Challenges of feeding a growing planet...

By 2030, it is estimated that we will need to feed a world population of 8.5 billion people. How can we do that in a way that is environmentally sustainable?

You have been asked to be a part of a group of experts to analyse and report on the problems associated with meat consumption and how it relates to deforestation. Once you have completed the sections below, you will need to create a presentation to summarize your findings. You will create either a short (2-minute max.) video presentation or a presentation using Prezi/Powerpoint that includes answers to the questions below, as well as summarizing the pros and cons of deforestation. Be sure to include at least two arguments each both for and against the results of Brazilian rainforest deforestation. Use a variety of sources, reference them in your report and indicate how you know the sources are reputable.

Part 1 – Worldwide meat consumption

There has been increasing pressure on farmers to produce meat (beef, pork, lamb, chicken and other types of meat that humans eat), which has undesirable effects on the environment.

According to the Food and Agriculture Organization of the United Nations, the annual world meat consumption in 1965 was 24.2 kg per capita (per person). This had increased to 41.3 kg per capita in 2015.

a Assuming a linear model, determine the equation for worldwide meat consumption per capita since 1965.

b Use your model to predict meat consumption per capita this year.

c Predict the meat consumption per capita in 2030.

d Discuss whether you think this is a realistic model.

e Do you think this increase in meat consumption is sustainable? Explain.

To put this in perspective, take a look at the following table, which compares meat consumption in developed and developing countries.

	1997 meat consumption (million tonnes)	Average annual increase in meat consumption since 1997 (million tonnes)
Developed countries	98	0.8
Developing countries	111	4.6

f Graph each set of data on the same axes. Assume a linear model for each.

g Determine the equation of each line.

h What does the slope of each line represent? Which slope is more concerning? Explain.

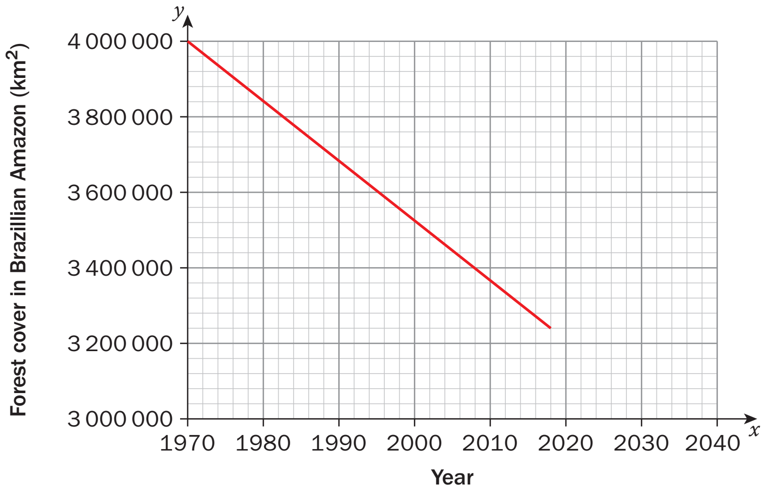
i Will there ever be a time when the meat consumption of developing countries is equal to that of developed countries? Explain. If it is possible, use your equations to try to find out approximately when this will happen.

You may want to think about the populations and their growth in developed and developing countries.

Part 2 – The Brazilian rainforest

Approximately 70% of clearcutting (cutting down all or most trees in an area) in the Brazilian rainforest is to provide land for cattle ranches and farming. This accounts for almost 15% of the world's total annual deforestation, making it the largest cause of deforestation worldwide.

The graph below represents the forest cover in the Brazilian Rainforest since 1970.



a Determine a linear equation that represents the relationship between the amount of forest cover in the Brazilian Amazon and the year. What does the y-intercept represent? What does the gradient represent?

b Assuming that the deforestation rate is constant, calculate the approximate area of forest that is cleared annually.

c To put this area in perspective, research the area of the city or town you live in in square kilometres. How does it compare with the area of the rainforest that is cleared each year?

d Given the calculated rate of area clearcut each year, how long does it take to clearcut an area of the rainforest that is the size of your town or city?

e Some scientists warn that the rainforest cannot deplete in size to less than 75% of its 1970 size without catastrophic irreversible consequences to the rainforest ecosystem. If clearcutting continues at the constant rate represented in the graph, in what year will that occur?

f Use a variety of sources and research three major reasons why we need the Brazilian rainforest. Reference your sources in your report and indicate how you know they are reputable.

Action plan

a Write an action plan containing at least five things that could be done to address the challenges of feeding a growing planet.

b Write down 3 personal decisions that you can make that will make a difference and describe their effects on the environment.

c Summarize your action plan and decisions in an interactive presentation using a program such as Prezi.